

Plate tectonic theory.

Types of plate boundaries (convergent, divergent, transform)

Elastic rebound theory

How earthquakes are generated?

(PT)

The earth's lithosphere is broken into about 12-15 major rigid plates (and several minor ones) that float on the semi-fluid asthenosphere.

These plates move due to convection currents in the mantle, driven by heat from the Earth's interior. The theory explains the distribution of earthquakes, volcanoes, and mountain ranges.

Convection currents in the mantle drive plate motion at rates of 1-10 cm/year.

Types of plate boundaries:

1) Convergent (Compressional) Boundary

Two plates move towards each other

One plate subducts beneath the other (subduction zone)

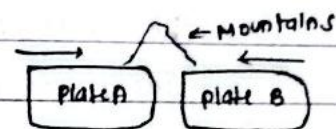
Oceanic plate subducts beneath oceanic plate

or Oceanic plate subducts beneath continental plate

Generates thrust/reverse fault earthquakes.

Eg: Nazca plate under South America.

Oceanic trench formed Collision b/w continental plates (mountain system)



2) Divergent (Tensional) Boundary

Two plates move apart (forming a gap)

spreading ridges

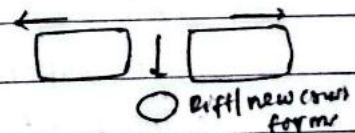
Less dense molten rocks from the mantle rise upward through gap

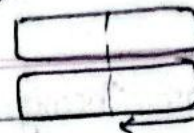
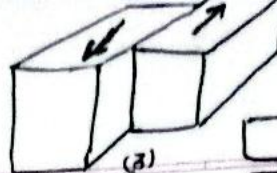
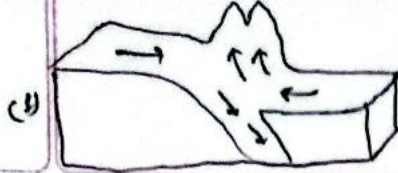
(cracks), solidifies and deposits new material onto the edges of

Oceanic plates, forming oceanic ridges $\Rightarrow$ sea-floor spreading.

Generates normal fault earthquakes

Eg: Mid-Atlantic Ridge





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Fault/Strike-slip
Major earthquake
hazard.

(3) Transform (Strike-slip) Boundary

- Two plates slide horizontally past each other, in opposite directions or same direction
- No crust is created or destroyed.
- Generates strike-slip earthquakes. (most damaging for eqs.)
- Eg. San Andreas fault

• Elastic Rebound Theory (Reid, 1906)

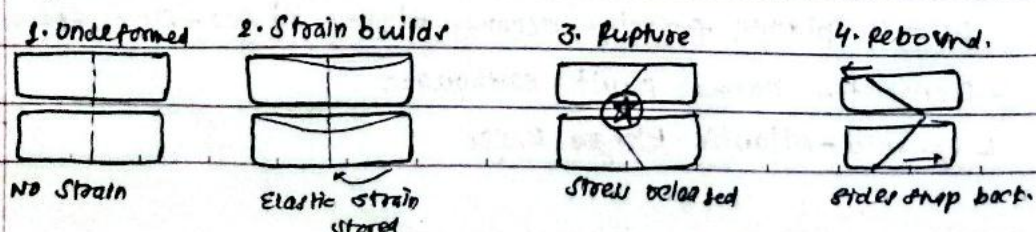
→ Based on observations after the 1906 San Francisco earthquake:

- Rocks on either side of a fault slowly accumulate elastic strain energy over decades/centuries as tectonic forces act on them.
- The fault remains locked due to friction, so strain builds up
- When the accumulated strain exceeds the frictional strength of the fault, sudden slip occurs
- The rocks on either side 'rebound' elastically to a new unstained position.
- This sudden release of elastic strain energy radiates outward as seismic waves → earthquake.

• How Earthquakes are Generated

- ↳ Tectonic plates move → stress accumulates at locked fault zones
- ↳ Stress exceeds rock strength → sudden rupture along the fault plane
- ↳ Seismic energy is released at the focus (hypocenter) - the point of rupture inside the Earth.
- ↳ The point directly above on the Earth's surface is the epicenter
- ↳ Seismic waves radiate outward in all directions from the focus.

Fig: ERT



⑧ Faults and fault mechanism

Normal, reverse/thrust, strike-slip, and oblique faults (with diagrams)

→ Fault is a fracture or zone of fractures in the Earth's crust along which rocks on either side have moved relative to each other. Egk. occurs due to sudden slip along this fault.

→ Fault mechanism (Egk. generation)

↳ Elastic rebound theory

↳ Rocks on either side of a fault accumulate elastic strain energy over time due to tectonic forces. When stress exceeds the frictional resistance of fault, sudden slip occurs releasing energy as seismic waves.

↳ Pt of initial rupture = Hypocenter / Focus

(9) PSHA vs DSHA		PSHA
Aspect	DSHA	PSHA
Approach	Worstcase scenario based	Probabilistic, all scenarios
Output	Single design ground motion	Hazard curve (probability vs intensity)
Uncertainty	Not explicitly handled	Explicitly quantified
Use	Critical facilities, dams	General building design
Return period	Not defined	Explicitly defined
Earthquake scenarios	Assumes single or a particular earthquake scenario	Assumes all possible earthquake scenarios
Focus	on maximum ground motion	on all possible ground-motion levels.
Distance	Select the closest distance b/w the source and site	Consider all possible distance b/w source and site.
Effects	Assume effects due to magnitude and distance	Consider all effects.
Approach	Focus on specific "maximum credible earthquake" (MCE) for a site	Consider all potential earthquakes from all relevant sources, accounting for their magnitude, distance & recurrence frequency.
Process	Identify seismic sources, select a maximum magnitude, and use ground motion prediction equations (GMPEs) to predict ground motion.	Integrate uncertainties in time, location, and size, providing a distribution of potential ground motions.
Pros/cons:	Clear and simple for engineering design (eg. P&A). It ignores the probability of such an event occurring, potentially leading to over-design.	Provides a more complete picture of the seismic risk. Complex to implement and depends heavily on the quantity of input data & the chosen GMPE.
Use case:	Critical structures where a specific, high-end scenario is needed	Setting building codes, insurance & risk assessment.

Poisson model: $1 - e^{-\lambda}$
 $\Rightarrow \left[T_R = \frac{-T}{\ln(1-P)} \right]$

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(Q) Attenuation laws - why used; list models; estimate PGA using given attenuation relationship at a given distance, calculate return period from probability of exceedance.

→ As seismic waves travel away from source, their amplitude (& thus ground motion intensity) decreases with distance. This decrease is called attenuation or ground motion decay.

→ Why attenuation law used?

↳ to predict ground motion parameters (PGA, PGV, spectral accel) at a site

↳ used in PSHA & DSHA

↳ Essential in seismic zonation & design spectrum development.

↳ Input for structural design codes

→ General form of attenuation eqn.

$$\ln(Y) = C_1 + C_2 M + C_3 \ln(R) + C_4 R + E$$

$Y \rightarrow$ Ground motion parameter

$M \rightarrow$ Magnitude

$R \rightarrow$ source to site distance

$E \rightarrow$ error term.

→ Attenuation models (Ground Motion Prediction Eqns)

	Model	Region	Notes
✓ 1.	Atkinson & Boore (1995, 2003)	Western N. America	widely used globally
✓ 2.	Campbell (1993, 2003)	Active tectonic regions	Includes soil depth
✓ 3.	Boore et al. (1999)	Shallow crustal eqs	used V _{s30}
✓ 4.	Sadigh et al. (1997)	Rock & soil sites	Different for rock/soil
✓ 5.	Next Generation Attenuation (NGA-West2)	Global	Most modern; multiple model
✓ 6.	IS 1893 empirical formula	Indian Subcontinent	Used in NGA - adjacent code

Factors considered in Attenuation law:

- ↳ Magnitude (M_w preferred)
- ↳ Distance (epicentral, hypocentral, surface distance)
- ↳ Site condition (rock, soil - often via V_{s30})
- ↳ Fault mechanism

Recurrence relation

(i) Gutenberg-Richter recurrence law

$$\log \lambda_m = a - bM$$

$\lambda_m \rightarrow$ mean annual rate of exceedence of magnitude M

$a =$ logarithm of no. of eqs with magnitude ≥ 0

$b =$ slope of distribution and controls the relative proportion of large to small eqs.

$$\text{Return period (T)} = \frac{1}{\lambda_m}$$

(ii) Poisson Model

$$P(N=n) = \frac{\mu^n e^{-\mu}}{n!} = \frac{(\lambda_m t)^n e^{-\lambda_m t}}{n!}$$

$N \rightarrow$ no. of occurrences of particular event during given time interval ' t '

$\propto$ Prob. of occurrence of at least one event in a period of time ' t ', (Probability of exceedence)

$$[P(N \geq 1) = 1 - P(N=0) = 1 - e^{-\lambda_m t}]$$

Attenuation relationship

$$\ln \text{PHA (gals)} = 6.74 + 0.859M - 1.80 \ln(R+2.5)$$

$R \rightarrow$ min² distance b/w source to site in km.

$$1 \text{ gal} = 0.01 \text{ m/s}^2$$

PHA $\rightarrow$ peak Hz. accel².

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(Q) Response spectrum - define, explain concept relationship among S_a, S_v, S_d (pseudo-accl², pseudo-rel., spectral displacement); how response spectrum curve is obtained; compare seismic coefficient vs response spectrum method.

→ How Response spectrum curve obtained?

1. Record ground motion

obtain accelerogram (accl² vs time) from seismograph at a site during an eqk.

2. Idealize a SDOF system

3. Apply ground accl² as base excitation to SDOF system

4. Solve eqⁿ of motion

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{u}_g(t)$$

5. Find max^m response (disp, vel, or accl²) for that specific period T .

6. Vary the period T from very short (rigid str) to very long (flexible str) and record peak response for each

7. Plot the result.

• X-axis $\rightarrow T$; Y-axis : S_a (or S_d, S_v)

8. This gives the response spectrum curve for that particular ground motion & damping ratio.

9. For design spectrum:

average/smooth multiple spectra from diff-eqk records.

∞ Shape of R S

• short period (rigid) $\rightarrow$ High accl², low disp.

• Medium " $\rightarrow$ Peak accl² zone (amplification)

• long " (flexible) $\rightarrow$ low accl², high disp.

Feature	Seismic coefficient Method (Static)	Response Spectrum Method (Dynamic)
Nature	Eqv. static analysis	Dynamic analysis
Base shear	$V = A_h \times W$ $C_A = \text{seismic coeff.}$	Based on modal analysis + SRS/CBC combination
Force distribution	Assumed linear (triangular)	Based on mode shape,
Accounts for dynamic behaviour?	No	Yes (uses natural periods & modes)
Building irregularity	Not captured	Captured thru Modal Analysis
Applicability	Regular low-rise	Irregular, tall, or Important structure.
Complexity	Simple, hand calculation	Requires computer analysis
Accuracy	Conservative / approx.	More accurate
Multiple modes	Only fundamental mode	Multiple modes considered
Code req.	Allowed for regular buildings	Mandatory for irregular tall buildings

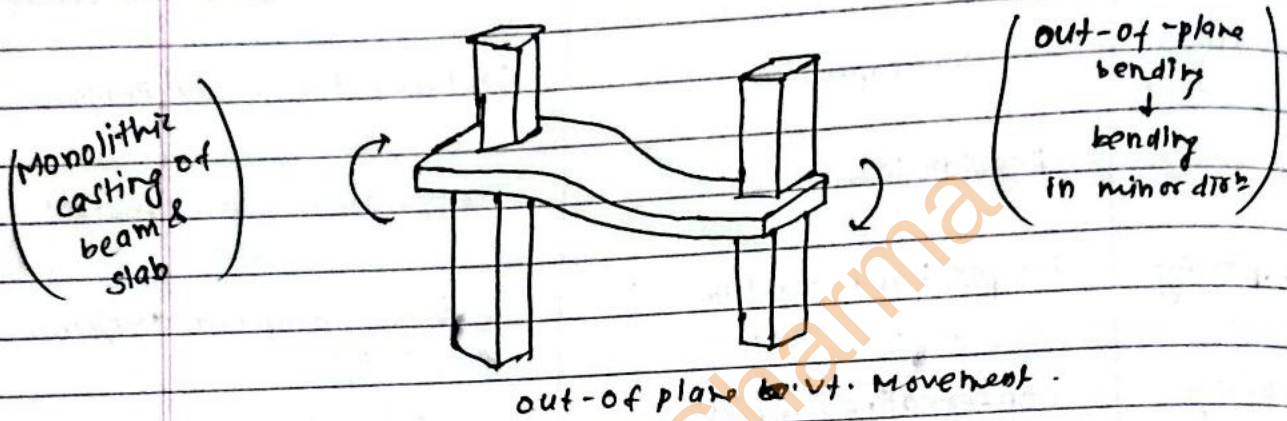
Feature
In plane stiffness
load distribution
eg
concrete

Rigid diaphragm
very high
by relative stiffness
RC slab, composite deck
significant

low
by tributary area
timber floor, light metal deck
negligible

(9) Rigid diaphragm concept

- Floor slabs are H_z. plate-like elements, which facilitate functional use of buildings. Usually beams and slabs at one story level are cast together.
- In residential multi-storey buildings, thk. of slab is only about 110-150mm. When beams bend in vt. direction during eqk., these thin slabs bend along with them.



- And, when beams move with columns in the h_z. direction, the slab usually forces the beams to move together with it. In most buildings, the geometric distortion of the slab is negligible. In h_z. plane, this behaviour is known as rigid diaphragm action.

→ In plane bending → column undergo deflection in H_z. movement

↳ In plane forces transfer from one end of building to other end of building (Rigid diaphragm action)

✓ Diaphragm is a h_z structural element (floor/roof) that transfers lateral (earthquake/wind) loads to vertical elements (shear wall, col., frame).

✓ Rigid diaphragm → infinitely stiff in its own plane

(It does not deform in-plane when subjected to lateral loads)

↳ All pts on the floor move together as a rigid body in plane

↳ Lateral load is distributed to vt. elements in proportion to their relative stiffness

↳ In-plane stiffness >> out-of-plane stiffness

↳ concrete slab with suff. thk.

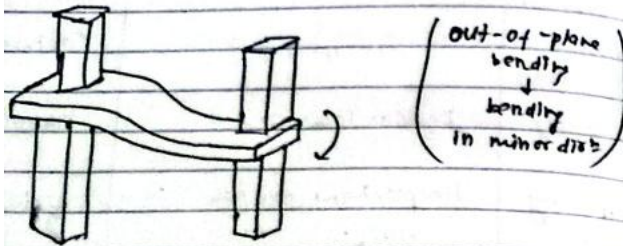
very high
By rigid slabs, composite deck
significant

by tributary area
timber floor, light metal deck
negligible

rigid concept

are H_z plate-like elements, which facilitate
re of buildings. Usually beam and slab at
level are cast together

a) multi-story building, thick of slab is only
150mm. when beam bend in vt direction
these thin slab bend along with them



out-of-plane & vt. movement

also move with column in the h_z direction,
ally forcing the beams to move together
most buildings, the geometric distortion
is negligible in h_z plane, this
is known as rigid diaphragm action.

ending → column undergo deflection in
H_z movement

↳ In plane force transfer from
one end of building to other end of
building (Rigid diaphragm action)

structural element (floor/roof) that transfer

a) Load to vertical elements (shear wall, col., frames).
indefinitely stiff in its own plane
does not deform in-plane when subjected to lateral
load
move together as a rigid body in plane
attributed to vt. elements in proportion to their

out-of-plane stiffness
vt. flex

Functional needs (asymmetry plan, re-entrant corner, varying stiffness)
Aesthetic choice (asymmetry plan, re-entrant corner, varying stiffness)
Structural decision (unequal column heights, varying stiffness)
Building Irregularities - types of plan & vertical irregularities (mass, stiffness, strength, geometry); how irregularities are developed; their effects on seismic performance; regular vs irregular buildings

Types

1. Plan (H_z) Irregularities

A. Traditional Irregularities

1. CM ≠ CR

→ Effects: corner columns & walls receive disproportionate force

2. Re-entrant corner

→ L, T, U, or + shaped building,
→ stress concentration at re-entrant corner
→ Effect: Differential movement b/w wings → cracking, separation

3. Diaphragm discontinuity

→ large opening, voids or sudden change in floor stiffness

→ Eff: Interrupted load path
↳ unpredictable force transfer

4. Out-of-plane offset

→ vt. elements (wall/columns) not aligned vertically

→ Eff: over turning, local failure

5. Non-parallel type

→ later force-susceptible elements not // to major axis

→ Eff: Bidiirectional coupling response

B. Vt. Irregularities

1. Soft storey / weak storey

→ one storey has significantly less stiffness or strength than adjacent storey
→ soft storey: stiffness < 70% of storey above
→ Eff: concentration of drift & ductility demands in soft storey → collapse

2. Mass Irregularity

→ Floor mass > 200% of adj. floor mass
→ Eff: Amplified seismic forces, higher mode effects

3. Geometric Irregularity (setback)

→ Plan dimension change significantly along h.
→ Eff: stress concentration at step, torsional effect

4. In-plane Discontinuity

- Shear on or column offset in plan at diff flow
- Eff.: overturning forces transferred thr. slab → local failure

5. Floating column

- col. doesn't continue to foundation (rests on beam)
- Eff. Extremely large row → high vt. load & no continuity → punching failure

⇒ Eff of irregularity

- Unpredictable load path
- concentration of stress/ductility demand.
- torsional amplification
- premature failure before energy dissipation
- code req. dynamic analysis for irregular buildings

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(A) Failure mechanisms of masonry structures - In-plane and out-of-plane failure modes; rocking, sliding, shear, toe crushing, diagonal tension; behaviour of masonry buildings during eqs.

→ Masonry is strong in comp., weak in tension & shear.

(A) In-plane failure (shear wall Action)

- Walls // to dirⁿ of seismic loading
- wall acts as shear wall

(1) Diagonal Tension / shear cracking

- ↳ X-shaped or diagonal cracks
- ↳ shear stress > tensile strength of masonry

(2) Sliding shear failure

- ↳ H₂ crack along bed joints (masonry-unit interface)
- ↳ shear stress > friction + cohesion along bed joints

(3) Rocking / Flexural failure

- ↳ V₁ crack at wall base / corner
- ↳ wall rocks about its corner (toe crushing)
- ↳ occurs in tall, slender walls with low axial load

(4) Toe-crushing

- ↳ comp. failure at compressed corner during rocking
- ↳ High axial load + overturning moment

(B) Out-of-plane failure

→ walls $\perp$ to dirⁿ of seismic loading

→ wall bends like a vertical slab under lateral force & cracking

(1) One-way bending

→ walls span vertically b/w floor & foundation

(2) Two-way bending

→ wall spans both vertically and horizontally b/w floors & cross walls

(3) Overturning - wall topples if not connected to top by diaphragm

critical factors

- ↳ slender wall ratio
- ↳ lack of connection to floor/roof
- ↳ Absence of the storey or vt. reinforcement.
- ↳ large unsupported panel area.

Surendra Sharma

Q) Strength of masonry in shear and flexure -
shear strength formula, factors affecting, flexural strength,
effect of vertical load; code provisions.

→ Shear strength of Masonry

Masonry resists shear thro-

- ✓ cohesion of mortar (c)
- ✓ friction along bed joints ($\mu\sigma$)
- ✓ interlocking of units

Mohr-coulomb failure criterion

$$\tau = c + \mu \cdot \sigma$$

τ ← shear strength μ ← coeff of friction
 c ← cohesion of mortar σ ← comp. stress normal to bed joints

Factor affecting

- ↳ Mortar type & quality
- ↳ Unit-mortar bond
- ↳ Aspect ratio of wall (H/L)
- ↳ presence of reinforcement or confinement

→ Flexural strength of Masonry

out of plane bending (more critical)

1. Flexural tensile failure (cracking)

$$\sigma_t = \left(\frac{P}{A} \right) - \left(\frac{M}{Z} \right) = f_t$$

wall cracks when net tensile stress $> f_t$

2 Compressive / toe crushing (in-plane rocking)

$$M_u = 0.5 * PL * \left(1 - \frac{P}{0.85 f'_m L t} \right)$$

P → axial load

L → wall length

t → " thickness

f'_m → comp. strength of masonry

Principles of Eqp. resistant design

Other key

- (1) Simple, regular configuration
 - ↳ Regular plan & elev.
 - ↳ Avoid torsion ($EM = 0$)
 - ↳ Minimize irregularity
 - ↳ Direct, cont. load path to base
- (2) Adequate lateral strength
 - ↳ Resist code-specified seismic force
- (3) Adeq. stiffness
 - ↳ Control drifts to protect non-str. elements
 - ↳ Prevent P- Δ instability
 - ↳ Balance flexibility & stiffness
- (4) Ductility & Energy dissipation
 - ↳ Str. should deform inelastically w/o collapse
 - ↳ by proper detailing, confinement
 - ↳ Capacity design (ensure ductile failure mode governs)
(flexure before shear, beam before column)
- (5) Redundancy
 - ↳ Multiple load paths $\rightarrow$ if one element fails, other carry load.
- (6) Capacity Design Philosophy
 - ↳ Strong column-weak beam
- (7) Adequate Foundation
 - ↳ Foundation must not fail before superstr.
 - ↳ Uniform settlement avoided.
- (8) Quality of construction
 - ↳ Proper detailing, cover, lap length, anchorage
 - ↳ on-site inspection
- (9) Soil-Structure interaction
 - ↳ Site amplification considered
 - ↳ Avoid building on soft soils w/o mitigation

③) SMRF vs OMRF

advantage of SMRF over OMRF

drift evaluation ~~mechanism~~ provisions.

Global stiffness matrix concept for 3D MRF.

Feature	SMRF	OMRF
Seismic design & ductility	Designed to <u>bend and dissipate energy without collapsing</u> , making them <u>suitable for high seismic zones</u> .	Designed for <u>low seismic risk</u> , with <u>lower ductility</u> and <u>reduced ability to handle severe energy dissipation</u> .
Detailing requirements	Requires <u>specific, intensive, ductile detailing</u> , such as <u>staggered reinforcement</u> in RC column and beam to prevent brittle failure.	Typically follows <u>general building codes</u> (Eg. IS-456) without specialized seismic requirements.
Cost & Material	<u>More expensive</u> due to higher steel quantities, specialized joints, and lighter spacing of shear reinforcement.	<u>less expensive</u> .
Structural performance	<u>provide better safety, lower base shear, and lower storey drift</u> during earthquakes.	<u>More economical and suitable for wind-loading</u> in low-seismic areas.
Code Application	According to seismic guideline like <u>IS 13920: 2016</u> , SMRFs are <u>mandatory for high seismic zone</u> .	OMRFs are <u>permitted only where low seismic forces are expected</u> .
Design	(IS 13920: 2016) → for design & detailing	(IS 456: 2000) → for design & detailing
Response reduction factor	<u>5</u>	<u>3</u>
Design shear	<u>High design base shear</u>	<u>Low design base shear</u>
Use	<u>For moderate to high eq.</u>	<u>For low earthquakes</u> .
Ductile check	<u>Safe to design a structure with ductile detailing</u>	<u>Not safe to design structure without ductile detailing</u> .

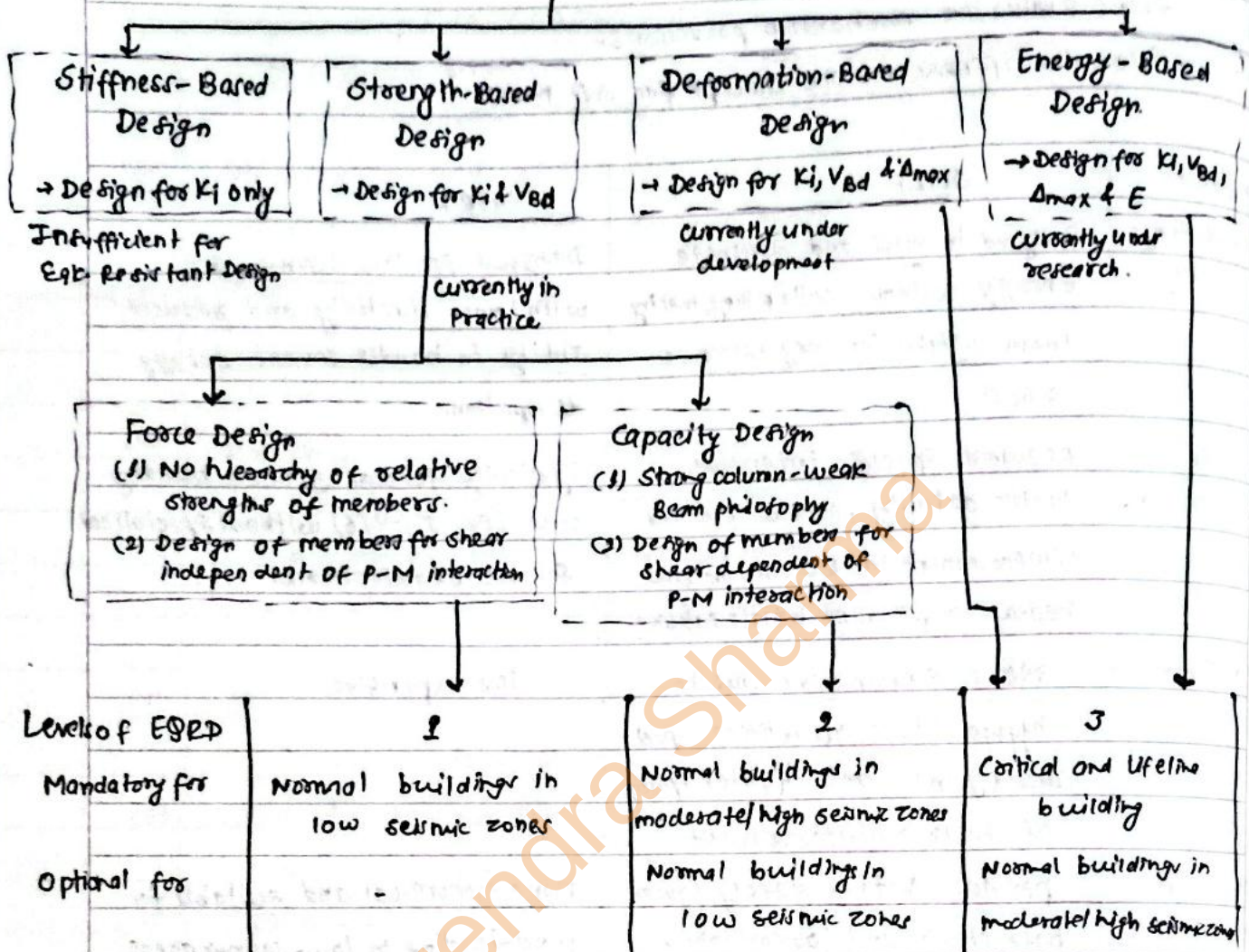
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NOTES

$K_i \rightarrow$ Stiffness
 $V_{ed} \rightarrow$ Strength
 $\Delta_{max} \rightarrow$ Ductility

P-M $\rightarrow$ Axial force - Bending Moment

Seismic Design



Current Earthquake Resistant Design Philosophies

Fig: Four broad methods available for Earthquake-Resistant Design;
 Rigorous increases in each higher level method.

FD design simply based on design lateral force on building; members are designed to resist the shear resultants obtained from linear structural analysis of the building subjected to code-specified design lateral forces.

No predetermined hierarchy of strengths across adjoining members and within each member.

? → why

FBD vs DBD

✓ Earthquake-resistant design aims to ensure safety and performance of structures under seismic loading. Two major approaches

(1) Force-Based Design (FBD)

↳ conventional method

↳ structures are designed to resist seismic forces calculated from design spectra

↳ based on lateral forces (bare shear)

↳ uses elastic analysis with force reduction factors (R)

↳ Inelastic behaviour is indirectly considered.

↳ widely used in building codes.

↳ does not directly control structural damage

↳ Assumes simplified behaviour

↳ Less accurate for severe earthquakes.

(2) Displacement-based Design (DBD)

↳ Modern approach

↳ structures are designed to achieve a target displacement or drift under seismic loading.

↳ Based on displacement or drift control.

↳ Directly ~~control~~ considers inelastic behaviour.

↳ Ensures specific performance levels (e.g. immediate occupancy, life safety).

↳ More realistic representation of structural response

↳ More complex analysis

↳ Requires advanced understanding.

- ✓ Force depends on stiffness
- ✓ But failure depends on deformation.

✓ Earthquake behaviour is better explained by Elastic Rebound Theory while Energy is released due to sudden fault movement, which causes ground displacement, not just force and displacement become a more reliable design parameter.

- $G_s \cdot \text{disp} = 0 \rightarrow$ No inertia force
- $\text{disp} = \text{large} \rightarrow$ structure must deform
- Egt. demand is fundamentally deformation demand.
- force depends on how structure responds
- But, same ground motion $\rightarrow$ different forces in different buildings!

Stress reversal: Dangerous?

→ Behavior (S)

Egt. $\rightarrow$ cyclic loading $\rightarrow$ tension $\rightarrow$ compression $\rightarrow$ tension $\rightarrow$ comp. $\rightarrow$...

→ In material like concrete

- ✓ Strong in compression
- ✓ weak in Tension

So during reversal

- ✓ cracks open $\rightarrow$ close $\rightarrow$ re-open.
- ✓ Reinforcement yields back & forth.

Leads to

- ↳ fatigue
- ↳ Bond failure
- ↳ stiffness degradation.

Thus,

- ✓ Repeated cycles $\rightarrow$ strength reduces even if peak load same.
- ✓ Hysteresis loops show energy dissipation but degradation.

Why Allow Damage in eqk. design?

(Why not elastic?)

→ Elastic design requires

$$F_{\text{elastic}} = k \cdot \Delta$$

To keep Δ small,

- k (stiffness) must be very high.
- $\rightarrow$ large member sizes $\rightarrow$ higher cost

→ Trade-off

Elastic design $\rightarrow$ safe but uneconomical

Inelastic design $\rightarrow$ controlled damage but economical.

→ Damage = energy dissipation mechanism

✓ w/o damage

↳ energy accumulator $\rightarrow$ sudden failure

✓ with ductility

↳ energy dissipater gradually

Why Ductility is most imp property?
→ Egt. Input = energy + displacement

Structure must

↳ Absorb energy

↳ Survive deformation.

→ Load deformation behaviour

• Brittle

↳ sudden drop after peak

↳ No warning

• Ductility

↳ Large deformation after yield

↳ Gradual failure.

→ Experimental behaviour

↳ Ductile member → stable hysteresis loop

↳ Brittle " → pinched loops → sudden drop

Thus

Ductility = ability to survive uncertainty

or

✓ Egt. magnitude unpredictable

✓ Deformation demand uncertain.

Simple Geometry: Better? (Geometry controls how force flows)

→ Egt. force travel as load paths.

Ideal case

st. path → uniform stress

Complex geometry

Load path bends → stress concentration

→ Egt. Convex shape → Direct force transfer

Concave shape → Forces must 'turn corners' → Bending + torsion.

→ Irregular buildings → torsional response

↳ stress concentrated at re-entrant corners.

Stiffness is critical? (Stiffness controls how much building moves)

→ Form:

$$\Delta = \frac{F}{k}$$

Lower stiffness → higher displacement

→ But

Egt. gives displacement demand, not force

so,

If structure is too flexible → large deformation → damage.

→ Flexible building

↳ Large drift ↳ Non-structural damage.

→ very-stiff building

↳ High force demand.

→ uneven stiffness → irregular behaviour.

→ soft story causes concentration of deformation

Why strength alone is not enough?

→ strength = ability to resist force.

But egk problem is:

↳ Not just force

↳ But deformation + energy.

→ Failure case:

↳ Strong but brittle building → collapse suddenly.

→ Exp Behaviour

↳ High str^t concrete w/o confinement → brittle.

↳ Moderate str^t with ductility → better performance.

→ Strength w/o ductility → dangerous illusion of safety.

Natural period?

→

$$T = 2\pi \sqrt{\frac{m}{k}}$$

✓ Every structure has a preferred vibration frequency

✓ Earthquake combines multiple frequencies.

→ Response spectrum shows:

certain periods → max. response

→ Design = avoid dangerous resonance regions

Taller Buildings: Larger period?

→ As ht. ↑ → mass ↑ & stiffness ↑ (longer columns)

$$T \propto \sqrt{\frac{m}{k}}$$

Thus, $T \uparrow$

→ Short buildings → stiff → small T

Tall " → flexible → large T

Why Infill wall change everything? (Infill is unintended structural element)

→ Infill acts like

↳ Diagonal compression strut

→ Effect:

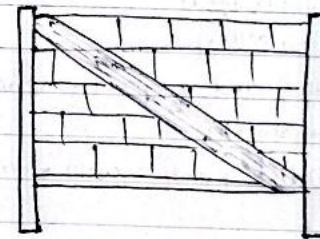
↳ Increases stiffness dramatically.

→ Exp.

Base frame → flexible }
Infilled frame → stiff. }

→ Danger

↳ Uneven infill → irregular stiffness → torsion.



← Infill wall acts as diagonal compression strut

(Infill wall ↑ stiffness and strength of frame)

- ✓ common in building with open ground floor
- ✓ during eqk deformation concentrates at this storey
- ✓ column in soft storey experience high bending and shear
- ✓ leads to formation of storey mechanism
- ✓ sudden collapse of that storey
- ✓ upper floors remain relatively undamaged
- # Soft storey: Dangerous? (must be avoided or strengthened)
- Soft storey = sudden drop in stiffness

$$\Delta = \frac{F}{k}$$

$$[k \downarrow \rightarrow \Delta \uparrow]$$

- ✓ deformation concentrates in one storey
- ✓ column fail there

=> Eqk damage localizer where stiffness is minimum.

Short column effects

→ Short column → higher stiffness

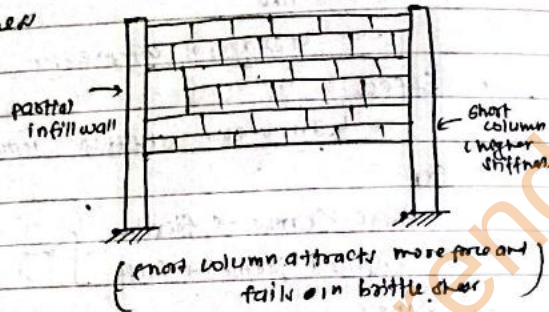
$$k \propto \frac{EI}{L^3}$$

$$[L \downarrow \rightarrow k \uparrow]$$

Attract more force

There

- ✓ Brittle shear failure
- ✓ Stiffer elements attract more force
- ✓ failure occurs in brittle shear mode.
- ✓ common in partially infilled frames
- ✓ cracks develop diagonally and propagate rapidly
- ✓ Failure is sudden & w/o warning
- ✓ Proper detailing & avoiding partial infill reduce this effect



Buildings don't fail because of force - they fail because they cannot deform safely under imposed displacement.

④ Four virtues of Eqk. Resistant Building

1) Seismic structural configuration

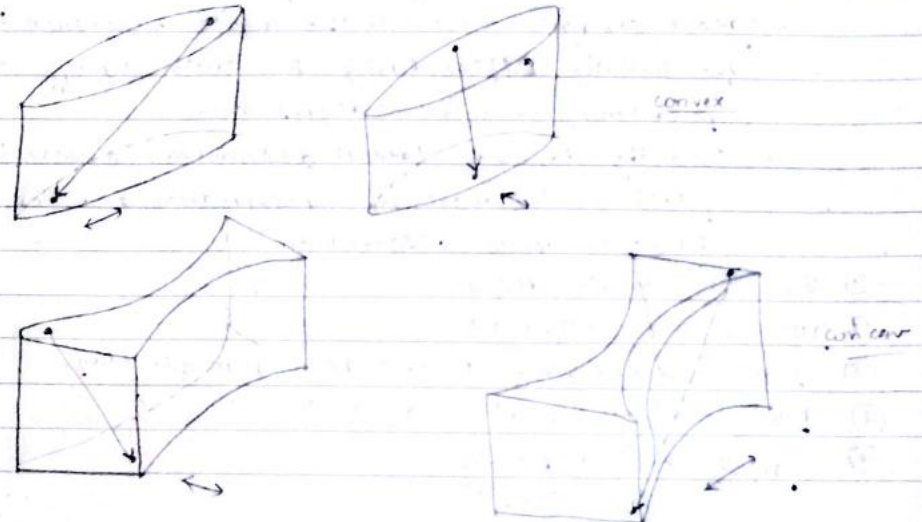
- (a) Geometry, shape & size of building
- (b) Location & size of structural elements
- (c) Location & size of significant non-structural elements

✓ Convex geometry preferred than concave geometry

↳ better eqk. performance

↳ direct load paths for transferring eqk. induced inertia forces to their bases

↳ concave: bending of load paths (stress concentration at corner)



✓ Rectangular plan & straight elevation

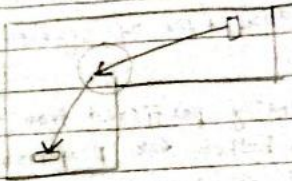
↳ perform well in eqk.

↳ internal force transferred w/o bend

✓ Buildings with setbacks & central opening

↳ offer geometric constraint to flow of inertia force

↳ inertia force paths have to bend before reaching ground



⇒ ✓ Lateral stiffness refers to the initial stiffness of building, even though stiffness of building reduces with increasing damage.

✓ Lateral strength refers to the max^m resistance that the building offers during its entire history of resistance to relative deformation.

✓ Ductility towards lateral deformation refers the ratio of the maximum deformation and the idealised yield deformation.

Thus, (Eqk. resistant design)

① Good structure configuration

② Min^m lateral stiffness in each of its plan directions.

③ Min^m lateral strength " " " " " "

④ Good overall ductility

① → no architectural form which becomes detrimental to good eqk performance.
no newer complexities.

② → no discomfort to occupants of building and no damage to contents of the building.

③ → to resist low intensity ground shaking with no damage, and not too strong to keep the cost of construction in check, along with a min^m vertical strength to be able to continue to support the gravity load and thereby prevent collapse under strong eqk shaking.

④ → to accommodate the imposed lateral deformation b/w the base and the roof of the building.

⇒ Both architect & engineer work together to create the best design with good interaction at all stages of the process of the design of the building.

Architect → form, functionality, aesthetic & contents.

Engineer → safety and desired eqk. performance.

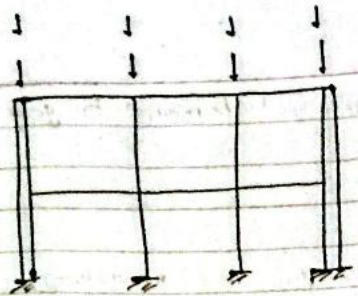
⇒ Achieve?

↳ by inputs provided in all stages of development of building in its planning, design, construction and maintenance.

✓ conceptual design

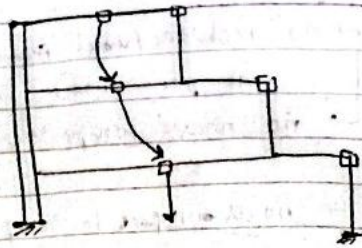
✓ design development thr. peer review of structural design

✓ construction thr. quality control and quality assurance procedure.



Good configuration

Direct, continuous load path



Bad configuration

Irregular load path
↳ stress concentration

✓ Simple and regular buildings perform better in ^{eqk.} buildings

Response spectrum? (Bridge b/w ground motion & structural response)

↳ Graph of max^m response v/s natural period.

↳ Eqk. motion = random combination of frequencies.

Each building

↳ has own natural period (T)

↳ 'pick' energy from eqk. matching its frequency.

↳ What spectrum does?

↳ Instead of solving time history for every building, we precompute
"If a structure had period T , what is the maximum response it would experience?"

↳ Some eqk.:

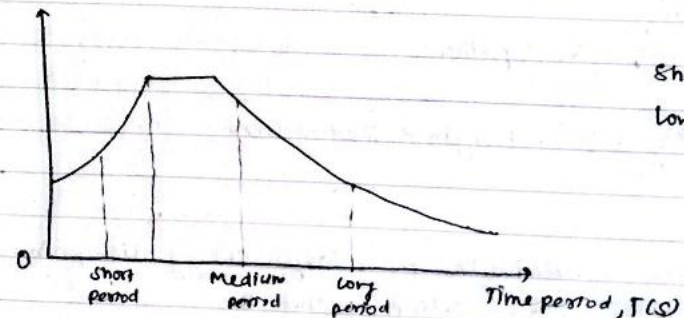
↳ Diff. Building → diff. responses.

Because, Response $\neq$ ground motion directly

↳ Exp

↳ Short-period buildings → high acceleration.
↳ Long-period buildings → high displacement.

Spectral
acc^{ls}
($\frac{a_g}{g}$)



Short T → high acc^{ls}

Long T → high disp.

Fig: Graph of max^m response of SDOF system for different natural periods
(Response spectrum)

Why accel, veh Disp spectra exist

→ Eqⁿ of motion:

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{u}_g$$

Acceleration → force demand

velocity → energy dissipation

Displacement → deformation demand

Response reduction factor (R):

→ R is not just a 'code factor'

→ From elastic to design force:

$$F_{\text{design}} = \frac{F_{\text{elastic}}}{R}$$

→ R?

1. Ductility

↳ ability to deform beyond yield

2. Overstrength

↳ Actual strength > design strength

3. Redundancy

↳ Multiple load paths

So,

$$R = \text{Ductility} \times \text{Overstrength} \times \text{Redundancy}$$

→ Exp. Behaviour,

↳ Ductile systems → stable hysteresis → large energy dissipation.

↳ Brittle systems → sudden failure → small R

⇒ R is not a reduction - It's a reward for a good behaviour

→ High R → more ductile str.

→ Brittle str. → low R values

→ R: allows economic design, allows controlled damage

→ Theoretical spectrum ($\frac{0.5g}{g}$) corresponds to elastic behaviour of structure.

But real structures are designed to yield and dissipate energy through ductility.

↳ Ductile structures can sustain large eqk. w/o collapsing, hence the elastic seismic force can be reduced by (R).
↳ considering ductility, overstrength and redundancy.

Energy concept [Eqk design = Energy management problem]

→ Eqk. Input energy into structure:

$$\text{Input} = \underbrace{E_{\text{elastic}}}_{\text{Recoverable}} + \underbrace{E_{\text{damping}}}_{\text{lost as heat}} + \underbrace{E_{\text{plastic}}}_{\text{Permanent change}}$$

↳ If structure cannot dissipate energy
↳ energy accumulates → failure

→ Exp.

↳ Ductile structures → large hysteresis loops → high energy dissipation.

↳ Brittle → narrow loops → low dissipation

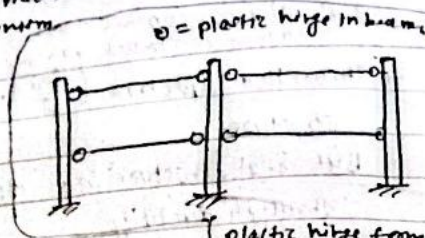
→ ~~energy accum~~

- ✓ column supports vertical loads → critical elements
- ✓ Beams are secondary load-carrying members
- ✓ Plastic hinge form in beam → stable mechanism
- ✓ Prevent formation of column mechanism

Strong column-weak beam?
→ to prevent collapse mechanism

→ weak column

↳ If column fail → vertical load path lost (collapse)



plastic hinge form in beam first, column remain elastic → safe mechanism

Weak Beam

↳ If Beams yield → structure stays (No collapse)

→ Exp

↳ Beam yielding → stable mechanism
↳ Column yielding → instability

=> columns = backbone (must never fail first)

Capacity Design (controlling failure)

→ we cannot prevent failure so we control where it occurs

→ Design some elements stronger & force others to yield first

→ Eg: ↳ Beam yield → columns remain elastic.

→ Behavior:

↳ Predictable failure pattern

↳ Energy dissipated in desired locations.

=> Good design = planned damage.

→ brittle failure
Shear failure: Dangerous?

→ Two failure types:

↳ Flexural → ductile
↳ Shear → brittle

→ Shear failure: No warning; sudden collapse

→ Behavior:

↳ Diagonal cracks → immediate failure

=> Always design shear stronger than flexure.

Open ground storey fails?

→ Upper floors:

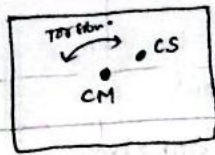
↳ Infill walls → high stiffness

Ground floor:

↳ No infill → low stiffness

Result: Drift concentrates at ground & columns at ground fail first

↳ Earthquake damage localized at weakest/stiffness zone.



CM & CS
do not
coincide
↓
Torsion

Torsion?

→ When

Center of mass $\neq$ center of stiffness

Then, Rotation occurs

→ Behavior:

- ✓ One side moves more } w
- ✓ Columns overloaded. } w

→ Asymmetry → Torsion → Uneven damage } w

Why mass distribution matters?

→ $[F = ma]$

→ Heavy floor $\Rightarrow$ more force } w

Uneven mass $\Rightarrow$ uneven force distribution. } w

→ Behavior:

Sudden heavy floor $\rightarrow$ failure points. } w

Soil affects response? (soil modifies eq. before it reaches structure)

→ Soil = spring + damper

→ Soft soil

- ↳ Amplifier motion } w
- ↳ Increased period } w

→ Hard soil

- ↳ Less amplification. } w

→ eg: Mexico city eq. $\rightarrow$ soft soil $\rightarrow$ severe damage

Why cracking changes behavior?

→ cracking reduces:

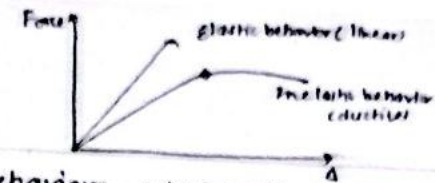
- ✓ Moment of Inertia } w
- ✓ Stiffness } w

Thus,

- ✓ Period increases } w
- ✓ Displacement increases. } w

→ Stiff $\rightarrow$ flexible transition during eq.

Earthquake Eng. is not about preventing motion,
but about ensuring safe deformation and controlled energy
dissipation. }



✓ Inelastic behaviour allows large deformation at nearly constant effort

Inelastic behaviour - what really happens after yield?

→ Structure goes beyond elastic → plastic deformation

→ Before yield,

$$F = k \cdot \Delta$$

After yield,

✓ stiffness drops drastically

✓ structure continues deforming with almost constant force
⇒ Structure sacrifices stiffness to survive.

→ Exp.

✓ Elastic: st. line (linear)

✓ Inelastic: curve flattens → large deformation

→ Why this is good?

↳ Allows large deformation w/o increasing force.

Hysteresis Behavior: (shape of hysteresis loop tells how safe the str. is)

→ Load-unload cycles during exp.

→ Exp.

↳ loop formed in force-deformation graph
↳ Area inside loop = energy dissipated.

→ Types of behavior:

1. Stable (ductile)

↳ wide loop

↳ high energy dissipation

2. Pinched loop

↳ reduced area

↳ less energy dissipation

3. Degrading loops

↳ strength reduces in each cycle.

Fig: Hysteresis loop (Energy dissipation)



↳ Area inside loop = Energy dissipated per cycle
↳ wider loop → more energy dissipation
↳ better ductility

Collapse Mechanism

→ Collapse happens when:

→ structure forms mechanism (unstable config.)

→ types

1. Beam Mechanism (Good)

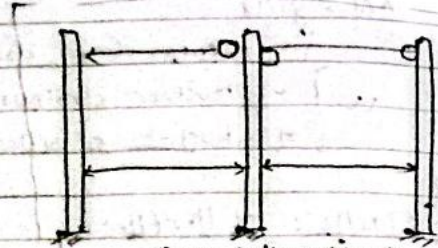
- ↳ Hinges in beams
- ↳ structure sways but stands

2. Column mechanism (Bad)

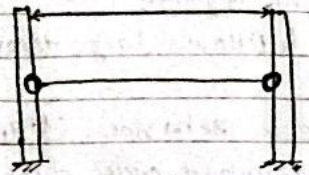
- ↳ Hinges in column
- ↳ vt load path lost → collapse

3. Storey mechanism (very bad)

- ↳ Hinges in one story
- ↳ soft storey collapse



Beam failure (safe)
Plastic hinges in beam
↳ stable structure



Column failure (dangerous)
Plastic hinges in column
↳ leads to collapse

⇒ collapse = loss of stability, not just strength.

- ⇒ must be considered in non-linear analysis
- ⇒ (significant in tall and slender buildings)
- ⇒ (amplification of lateral deformation)

P-Δ effect (secondary moment due to displacement)

→ When building deforms:

- ↳ vertical load (P) acts on displaced position (Δ)
- creates additional moment.

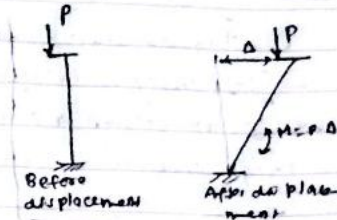
$$M = P \Delta$$

- More disp. → More moment → More disp.
positive feedback ⇒ instability

→ Behavior:

- ↳ sudden increase in drift
- ↳ collapse w/o warning

⇒ P-Δ effect turns stable structures into unstable system.



Higher mode effects:

- Buildings don't vibrate in one mode only
- ↳ Multiple mode shapes exist

→ For short buildings

- ↳ Fundamental mode dominates

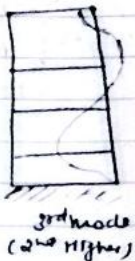
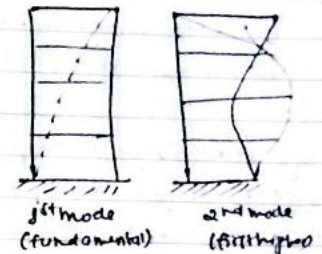
→ For tall buildings

- ↳ Higher modes significant

→ Behavior:

- ↳ Irregular deformation along ht.
- ↳ Force concentration in upper floor

⇒ tall buildings require multi-mode consideration.



→ Buildings vibrate in diff. modes during eqk

Why Damage localizer?

- Structure is not perfectly uniform.
- weakest location attracts deformation

$$\Delta = \frac{F}{k}$$

- Lower stiffness → higher deformation
- Soft storey → collapse at that level
- Weak column → failure there
- Soft floor and exploits the weakest link.

Redistribution of force

- After yielding,
 - ↳ stiffness changes
 - ↳ load path changes
- Forces redistribute to stronger members.
- Structure adapts during eq. (if ductile)

① # Why Redundancy is imp?

- Redundancy = multiple load paths.
- If one member fails → other carry load.
- Redundant str → gradual failure
- non-redundant → sudden collapse
- Redundancy = backup system for safety

Why overstrength exists

- Actual strength > design strength due to:
 - ✓ Material variation
 - ✓ Conservative design
 - ✓ Strain hardening
- Result: structure stronger than expected
- Overstrength provides hidden safety margin

Why Plastic Hinges form at ends

- Moment is max. at Beam-column joints
- So, yielding occurs there
- Behavior:

Rotation concentrated at ends

- Plastic hinge = energy dissipation zone

Confinement reinforcement (ductility booster)

- Concrete under compression → Expands laterally
- W/O confinement:
 - ↳ cracks → crushes.
- With confinement:
 - ↳ Steel restrains expansion → increases ductility
- Confined concrete → large strain before failure

- # Buildings do not collapse because they are weak,
 - ↳ but because they lose stability due to uncontrolled deformation and poor energy dissipation.

Why Eqs design allows damage?

- Eqs induces displacement-controlled loading, not force controlled.
- Designing str. to remain fully elastic requires very high stiffness and strength, making it uneconomical.
- Eqs input energy must be dissipated by the structure.
- Controlled damage (inelastic behavior) acts as a mechanism for energy dissipation.
- Elastic str. cannot dissipate sufficient energy → risk of sudden failure.
- Design philosophy ensures:
 - Minor EQ → no damage
 - Moderate EQ → repairable damage
 - Severe EQ → no collapse
- Allowing damage reduces design force using response reduction factors.
- Ductile behavior ensures gradual failure instead of brittle collapse.
- This approach balances safety and economy.
- Hence, eqs design is resistant, not earthquake-proof.

Imp. of ductility in eqs-resistant design.

- Ductility → undergo large def. w/o failure.
- Eqs → large displacement demand.
 - Brittle → no warn
 - Ductile → absorb & dissipate energy through plastic deformation.
 - redistribution of forces after yielding.
 - prevents sudden collapse by providing deformation capacity.
- Str. with high duct. → designed for lower forces (high R factor).
- Ductile member → stable hysteretic loop.
- life safety under severe shaking.

Role of Infill walls in seismic behavior

→ Infill walls increase lateral stiffness of frame

→ Act like diagonal compression strut

→ Reduce natural period of building.

→ Dec. lateral displacement

→ Uneven distribution causes stiffness irregularity.

→ Leads to torsion in building.

→ Absence at ground floor causes soft storey.

→ Increase force demand on structural members

→ Often ignored in design → unsafe assumptions.

→ Proper modelling improves accuracy of analysis.

Name: Surendra Sharma

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Subject: Earthquake Resistance Design of Structures
(CE 76501)

(Assignment)

[Q.1.10] Explain plate tectonic theory. Mention different types of magnitudes.

→ Plate tectonic theory:

The Earth's lithosphere is divided into several large and small rigid plates that float on the semi-fluid asthenosphere beneath them. These plates are in constant slow motion, driven primarily by convection currents in the mantle. The theory explains the distribution of earthquakes, volcanoes, and mountain ranges across the globe.

- The lithosphere is broken into about 12 major tectonic plates (e.g. Indian plate, Eurasian plate, Pacific plate, etc)
- Plates move at rates of 2-10 cm per year
- In Nepal's context, the Indian plate is subducting beneath the Eurasian plate at roughly 4-5 cm/yr, which is primary cause of Himalayan earthquakes including the 2015 Gorkha Earthquake (Mw 7.8)

Three types of plate boundaries:

(1) Convergent boundary

- ↳ plates collide
- ↳ e.g. Nepal/Himalayan region (Indian plate pushing into Eurasian plate)

(2) Divergent boundary

- ↳ plates move apart
- ↳ e.g. Mid-Atlantic Ridge

(3) Transform boundary

- ↳ plates slide past each other horizontally
- ↳ e.g. San Andreas Fault, California

→ types of Magnitudes

(i) Local/Richter Magnitude (M_L)

$$M_L = \log_{10} \left(\frac{A}{A_0} \right)$$

↳ Based on amplitude recorded on Wood-Anderson seismograph at 100km

(ii) Surface wave Magnitude (M_s)

$$M_s = \log_{10} \left(\frac{A}{T} \right) + 1.66 \log_{10} \Delta + 3.30$$

↳ For shallow earthquakes using surface waves, period ($T = 2\pi\omega$)

(iii) Body wave Magnitude (M_b)

$$M_b = \log_{10} A - 1.66 \log_{10} T + 0.01 \Delta + 5.9$$

↳ Uses P-waves, good for deep earthquakes.

(iv) Moment magnitude (M_w)

$$M_w = \frac{2}{3} [\log_{10} M_0 - 16]$$

($M_0 \rightarrow$ dyne cm)

↳ Most reliable, based on seismic moment; no saturation.

[Q.1 (b)]

Estimate the seismic moment and Moment magnitude of an earthquake

→ Solution:-

Given:

Strike slip Displacement, $D = 2.6$ m

Fault length, $L = 90 \text{ km} = 90,000 \text{ m}$

Fault depth (width), $W = 30 \text{ km} = 30,000 \text{ m}$

Rupture strength (shear stress), $\sigma = 160 \text{ kPa}$
 $= 160,000 \text{ N/m}^2$

Now,

$$\text{Rupture area (A)} = L \times W = 90,000 \times 30,000 = 2.7 \times 10^9 \text{ m}^2$$

$$\text{Seismic moment } M_0 = \mu \times A \times D = 160,000 \times 2.7 \times 10^9 \times 2.6 = 1.123 \times 10^{15} \text{ N m}$$

$$M_w = \frac{2}{3} [\log_{10} M_0 - 16]$$

$$(M_0 = 1.123 \times 10^{15} \times 10^7 \text{ dyne cm})$$

$$= \frac{2}{3} [\log_{10} (1.123 \times 10^{22}) - 16]$$

$$[M_w = 4.033]$$

[Q.2 (a)] Explain different ground motion parameters.

→ Ground motion parameters describe the characteristics of earthquake shaking at a site. Key parameters are:

1. Peak Ground Acceleration (PGA)

↳ Max² acceleration of the ground during shaking

↳ Unit: g or m/s^2

↳ Most commonly used in seismic design codes (NBC 105)

↳ Directly related to inertial force on structures.

2. Peak Ground Velocity (PGV)

- ↳ Max $\approx$ velocity of ground motion
- ↳ Better indicator of damage to medium-period structures.
- ↳ More relevant to pipelines and underground structures,

3. Peak Ground Displacement (PGD)

- ↳ Max $\approx$ displacement of ground
- ↳ Important for long-period structures (bridges, tall buildings)

4. Duration of shaking

- ↳ Longer duration causes more energy dissipation and cumulative damage.
- ↳ Significant bracketed duration: time b/w first and last exceedance of 0.05g

5. Frequency content / Power Spectral Density

- ↳ Describes how energy is distributed across frequencies
- ↳ Soft soils amplify low-frequency (low-period) content.
- ↳ In Nepal, Kathmandu valley's soft lacustrine soil significantly amplifies ground motion (as observed in 2015)

6. Response Spectrum

- ↳ Plot of maximum response (acceleration, velocity, displacement) of SDOF systems vs natural period for a given damping ratio.
- ↳ Foundation of modern seismic design (NBC 105:2020) provides response spectra for Nepal

[Q. 2-(b)]. Explain how probabilistic seismic hazard analysis is carried out.

→ Probabilistic Seismic Hazard Analysis (PSHA)

PSHA estimates the probability that the ground motion will exceed a specified level at a site during a given time period. It combines uncertainties in earthquake location, magnitude and ground motion prediction.

Steps in PSHA

1. Identify and characterize seismic sources
 - ↳ Identify all potential earthquake sources (faults, areas) around the site; prob distribution of source-site distances; source geometry
 - ↳ In Nepal: Main Central Thrust (MCT), Main Boundary Thrust (MBT), Main frontal thrust (MFT) are major sources

2. Characterization of seismicity of each source zone

- ↳ Also called Recurrence relationship
- ↳ Use Gutenberg-Richter law:
$$\log N = a - bM$$

- ↳ Determine the rate of earthquakes of various magnitudes from historical data.

3. Determination of motion from each source zone

- ↳ Ground motion prediction (Attenuation)
- ↳ Use attenuation / GMPE equations to estimate ground motion (eg. PGA) at the site for given magnitude and distance
- ↳ Eg. $\ln(PGA) = C_1 + C_2 M + C_3 \ln(R) + \dots$
- ↳ Account for uncertainty σ using standard deviation (σ)

4. Compute Probability of Exceedance

↳ For each source, integrate over all possible magnitudes and distances.

$$L_1(Y|S) = \sum v_i \iint P(Y>S|m, r) \cdot f_m(m) \cdot f_r(r) \cdot dr \cdot dm$$

Finally, develop Hazard curve.

↳ plot annual probability of exceedance vs ground motion level

↳ Form hazard curve, extract ground motion for desired return period.

[Q. 3 (a)]

Given,

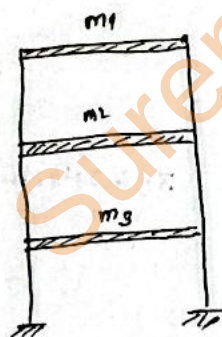
3-storey building

$$[m] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ kip-s}^2/\text{ft}$$

$$[\Phi] = \begin{bmatrix} 1 & 1 & 1 \\ 1.648 & -0.60 & -2.26 \\ 0.2 & -0.67 & 2.1 \end{bmatrix}$$

$$\{u(t)\} = \begin{Bmatrix} 1.56 \\ -0.5 \\ 0.75 \end{Bmatrix} \text{ ft/s}$$

$$\{w\} = \begin{Bmatrix} 4.83 \\ 9.15 \\ 14.31 \end{Bmatrix} \text{ rad/s}$$



Assume, ht. of each storey = 10ft

$$\text{Mass matrix } [m] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ kip-s}^2/\text{ft}$$

Mode shapes

$$\{\Phi_1\} = \begin{Bmatrix} 1 \\ 1.648 \\ 0.2 \end{Bmatrix}, \{\Phi_2\} = \begin{Bmatrix} -0.60 \\ -0.67 \end{Bmatrix}, \{\Phi_3\} = \begin{Bmatrix} 1 \\ 2.26 \\ 2.1 \end{Bmatrix}$$

Modal participation factors:

$$\lambda_n = \frac{L_n}{M_n} = \frac{\{\Phi_n\}^T [m] \{1\}}{\{\Phi_n\}^T [m] \{\Phi_n\}}$$

$$L_1 = \{\Phi_1\}^T [m] \{1\}$$

$$= \begin{bmatrix} 1 & 1.648 & 0.2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$= 3.872 \text{ kips-s}^2/\text{ft}$$

$$M_1 = \{\Phi_1\}^T [m] \{\Phi_1\}$$

$$= \begin{bmatrix} 1 & 1.648 & 0.2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1.648 \\ 0.2 \end{Bmatrix}$$

$$= 5.538 \text{ kips-s}^2/\text{ft}$$

Similarly,

$$L_2 = -1.24 \text{ kips-s}^2/\text{ft}$$

$$M_2 = 2.4378 \text{ kips-s}^2/\text{ft}$$

$$L_3 = 1.81 \text{ kips-s}^2/\text{ft}$$

$$M_3 = 17.4814 \text{ kips-s}^2/\text{ft}$$

$$\lambda_1 = \frac{L_1}{M_1} = 0.751$$

$$\lambda_2 = \frac{L_2}{M_2} = -0.509$$

$$\lambda_3 = \frac{L_3}{M_3} = 0.103$$

3 Storey displacements:
 $\{u_n(t_1)\} = \{ \phi_n \} \frac{L_n}{M_n} \cdot \frac{v_n(t_1)}{\omega_n} \rightarrow \text{Modal displacement}$

Mode 1:
 $\{u_1(t_1)\} = \begin{Bmatrix} u_{11}(t_1) \\ u_{21}(t_1) \\ u_{31}(t_1) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 3.648 \\ 0.2 \end{Bmatrix} \times 0.751 \times \frac{1.56}{4.88}$
 $= \begin{Bmatrix} 0.24 \\ 0.396 \\ 0.048 \end{Bmatrix} \text{ ft.}$

Mode 2:
 $\{u_2(t_1)\} = \begin{Bmatrix} u_{12}(t_1) \\ u_{22}(t_1) \\ u_{32}(t_1) \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0.60 \\ -0.67 \end{Bmatrix} \times -0.509 \times \frac{-0.5}{9.15}$
 $= \begin{Bmatrix} 0.028 \\ -0.017 \\ -0.019 \end{Bmatrix} \text{ ft.}$

Mode 3:
 $\{u_3(t_1)\} = \begin{Bmatrix} u_{13}(t_1) \\ u_{23}(t_1) \\ u_{33}(t_1) \end{Bmatrix} = \begin{Bmatrix} 1 \\ -2.26 \\ 2.1 \end{Bmatrix} \times 0.103 \times \frac{0.75}{14.31}$
 $= \begin{Bmatrix} 0.005 \\ -0.012 \\ 0.011 \end{Bmatrix} \text{ ft.}$

$\therefore$ Total displacement at time (t_1)
 $\{u(t_1)\} = \{u_1(t_1)\} + \{u_2(t_1)\} + \{u_3(t_1)\}$
 $= \begin{Bmatrix} 0.273 \\ 0.367 \\ 0.04 \end{Bmatrix} \text{ ft.}$

4 Lateral force at each storey:

$$\{f_s(t_1)\} = [m] [\phi] \left\{ \frac{L_n}{M_n} \omega_n v_n(t_1) \right\} \rightarrow \text{total force}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1.648 & -0.60 & -2.26 \\ 0.2 & -0.67 & 2.1 \end{bmatrix} \begin{Bmatrix} 0.751 \times 4.88 \times 1.56 = 5.72 \\ -0.509 \times 9.15 \times 0.5 = -2.33 \\ 0.103 \times 14.31 \times 0.75 = 1.10 \end{Bmatrix}$$

$$= \begin{Bmatrix} 9.15 \\ 8.31 \\ 3.78 \end{Bmatrix} \text{ kips.}$$

5 Shear force at each storey.

$$\{V(t_1)\} = \begin{Bmatrix} 9.15 = 9.15 \\ 9.15 + 8.31 = 17.46 \\ 9.15 + 8.31 + 3.78 = 21.24 \end{Bmatrix} \text{ kips.}$$

6 Overturning moment at each storey

$$M_{01}(t_1) = 10 \times 9.15 = 91.5 \text{ kips-ft (upper storey)}$$

$$M_{02}(t_1) = 20 \times 9.15 + 10 \times 8.31 = 266.1 \text{ kips-ft (Mid-storey)}$$

$$M_{03}(t_1) = 30 \times 9.15 + 20 \times 8.31 + 10 \times 3.78 = 478.5 \text{ kips-ft (Base storey)}$$

[9.3(b)] Explain behaviour of masonry building during failure.

→ Behaviour of masonry building during failure:

Pre-Earthquake Behaviour:

Masonry buildings carry predominantly vertical (gravity) loads through compression. Masonry building is strong in compression but very weak in tension and shear. Under gravity loads alone, masonry performs adequately.

When Eqs. strike:

Stage 1 - Initial cracking:

- ↳ Lateral inertial forces are generated proportional to the mass of the building.
- ↳ Horizontal shear force induced in walls.
- ↳ First cracks appear at weakest points - Openings (doors, windows), corners, and wall junctions.

Stage 2 - Crack Propagation:

- ↳ Diagonal tension crack propagate across wall panels (X-shaped or stepped cracks along mortar joints).
- ↳ Walls 1x to shaking direction begin to bow outward (out-of-plane bending).
- ↳ Loss of connection b/w perpendicular walls becomes apparent.

Stage 3 - Partial collapse:

- ↳ Out-of-plane walls (gable ends, parapets) topple outward first (most common cause of casualties in Nepal).
- ↳ Corner regions separate as interlocking is poor in traditional construction.

Stage 4 - Progressive/Total collapse:

- ↳ Once walls collapse - roof/floor lose support & fall.
- ↳ In rubble stone masonry buildings (common in rural Nepal), complete pancake collapse occurs.

Key Behavioural characteristics

- No warning - collapse is sudden and brittle, unlike RC structures which show ductile deformation.
- Lack of box action - walls act independently rather than as a unified box.
- Bi-directional vulnerability - both in-plane and out-of-plane failure can occur simultaneously.
- Heavy self-weight - generates large inertial forces.

[9.4(a)]

→ Given 2-D frame (1-storey)
rigid diaphragm
uniformly distributed floor mass
Mass of roof $\rightarrow 1500 \text{ kg/m}^2$
 $E Q_x = 300 \text{ kN}$

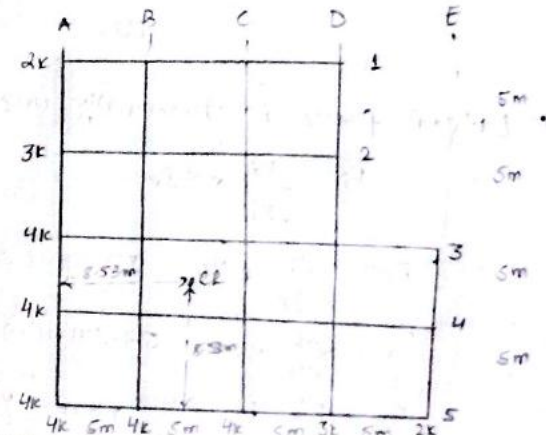
* CM

$$x_m = \frac{\sum A_i \bar{x}_i}{\sum A_i}$$

$$= \frac{20 \times 20 \times 10 - (5 \times 10) \times (17.5)}{20 \times 20 - 5 \times 10}$$

$$= 8.93 \text{ m}$$

$$y_m = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{(20 \times 20) \times 30 - (5 \times 10) \times (10)}{20 \times 20 - 10 \times 5} = 9.28 \text{ m}$$

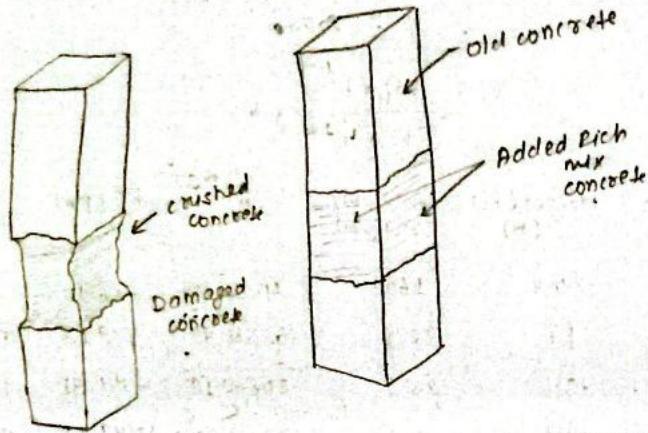


[9.5.16]

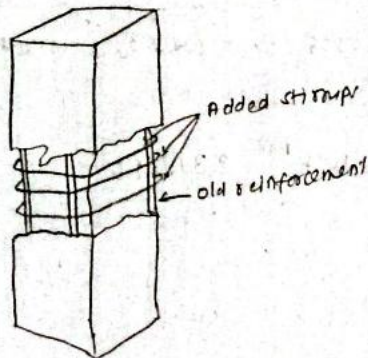
→ various methods of retrofitting of RCC structures

③ Rich mix concreting overlay

↳ removing the cracked concrete in the area and re-concreting with rich mix of concrete



④ Retrofitting of column by adding stirrups/reinforcement



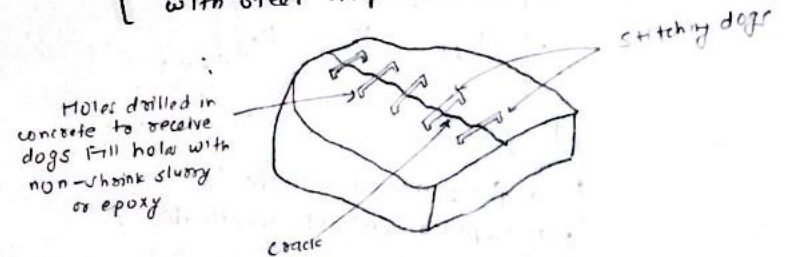
⑤ Spalling of concrete with buckling of bar

④ Floor Damage

↳ Simple rich concrete & mortar overlay
↳ RCC slab repairs by adding reinforcement & dowels bars

⑤ Other crack repair techniques

↳ Grouting and micro-concreting
(Repair of minor cracks in RC members)
↳ Routing and Sealing
(enlarging the cracking along its exposed face and sealing it with a suitable joint sealant or rich mixed mortar)
↳ stitching
(Drilling holes on both sides of the crack)
(Grouting in U-shaped metal units with short legs called staples or stitching dogs or bolting with steel strips on both sides of crack)



(c)
$$[K] = \begin{bmatrix} (k_1+k_2) & -k_2 & 0 \\ -k_2 & (k_2+k_3) & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$$

(b) Concept of regular buildings for earthquake resistant design

• Symmetry

↳ The buildings as a whole or its various blocks should be kept symmetrical about both the axes. Asymmetry leads to torsion during earthquakes and is dangerous.

↳ Symmetry is also desirable in the placing and sizing of doors and window openings, as far as possible.

• Regularity

↳ Simple rectangular shapes, behave better in an earthquake than shapes with many projections.

↳ Torsional effects of ground motion are pronounced in long narrow rectangular blocks. Therefore, it is desirable to restrict the length of a block to three times its width.

↳ If longer lengths are required two-separate blocks with sufficient separation in between should be provided.

↳ Separation of large building into several blocks may be required so as to obtain symmetry and regularity of each block.

↳ For preventing hammering or pounding damage between blocks a physical separation of 3 to 4 cm throughout the height above the plinth level will be adequate as well as practical for upto 3 storeyed buildings.

Q.1(a) Describe plate tectonic theory. Explain different types of seismic waves

→ Plate tectonic theory states that Earth's outer shell (lithosphere) is divided into several large and small rigid plates that float and move over the semi-molten layer below (asthenosphere).

It is important as:

- (i) Most earthquakes occur at plate boundaries
- (ii) Movement of plates causes stress buildup and sudden release.
- (iii) It is the reason of rise of Himalayas

Types of seismic waves

(A) Body waves (travel through Earth's interior):

(i) P-waves

↳ Primary / compression waves

↳ Particle motion is parallel to wave propagation direction

↳ Fastest seismic waves; $v_p \approx 5-8 \text{ km/s}$ in crust

↳ Travel through solids, liquids, and gases

↳ Fastest to arrive at seismograph.

(ii) S-waves

↳ Secondary / shear waves

↳ Particle motion is perpendicular to propagation direction

↳ $v_s \approx 3-5 \text{ km/s}$ (roughly $v_p/\sqrt{3}$ in elastic medium)

↳ Travel only through solids

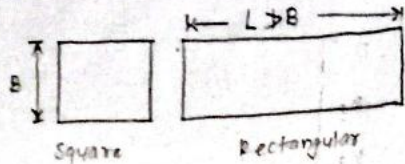
↳ More destructive than P-waves.

↳ Velocity relationship:

$$\frac{v_p}{v_s} = \sqrt{\frac{\lambda + 2\mu}{\mu}}$$

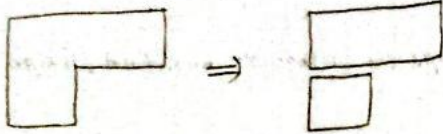
for Poisson's ratio = 0.25

$$\frac{v_p}{v_s} \approx \sqrt{3}$$



eg:

L-shaped:



U-shaped:



(B) Surface waves (travels along Earth's surface):

(i) Love waves

↳ Horizontal shear motion, no vertical component

↳ Travel faster than Rayleigh waves

↳ Cause significant horizontal damage.

(ii) Rayleigh waves

↳ Elliptical retrograde particle motion

(both horizontal and vertical)

↳ Slower than body waves but larger magnitude

↳ Cause the characteristic "rolling" ground motion

↳ Most destructive surface wave.

Approximately

$$V_p > V_s > V_{\text{rayleigh}} > V_{\text{love}}$$

In general,

$$V_p \approx 1.73 V_s$$

$$V_{\text{rayleigh}} \approx 0.92 V_s$$

[Q. 1(b)]

An earthquake causes an average of 3m strike slip displacement over 100km long, and 40km deep of a transform fault. Assuming the rock along the fault had average rupture strength of 180kPa, estimate the seismic moment and moment magnitude of an earthquake.

→ Solution:

$$D = 3\text{m}$$

$$L = 100\text{km} = 10^5\text{m}$$

$$W = 40\text{km} = 4 \times 10^4\text{m}$$

$$\text{Rupture strength, } \mu = 180\text{kPa} = 1.8 \times 10^5 \text{ N/m}^2$$

Now,

$$\text{Rupture area, } A = L \times W = 10^5 \times 4 \times 10^4 = 4 \times 10^9 \text{ m}^2$$

$$\begin{aligned} \text{Seismic moment, } M_0 &= \mu \times A \times D = 1.8 \times 10^5 \times 4 \times 10^9 \times 3 \\ &= 2.16 \times 10^{15} \text{ N}\cdot\text{m} \\ &= 2.16 \times 10^{22} \text{ dyne}\cdot\text{cm} \end{aligned}$$

Moment Magnitude (M_w)

$$M_w = \frac{2}{3} [\log_{10} M_0 - 16]$$

$$= \frac{2}{3} [\log_{10} (2.16 \times 10^{22}) - 16]$$

$$[M_w = 4.22]$$

[Q No. 2(a)]

Explain liquefaction with schematic sketch and its causes.

→ ~~Explain~~
 ↳ Liquefaction is a phenomenon where saturated loose granular soil (typically sand) temporarily loses its shear strength and behaves like a liquid during earthquake shaking.

↳ During cyclic shaking, pore water pressure in saturated loose sand builds up. When excess pore pressure equals the effective confining stress, effective stress drops to zero and the soil loses all shear strength.

↳ Effective stress: $\sigma' = \sigma - u$

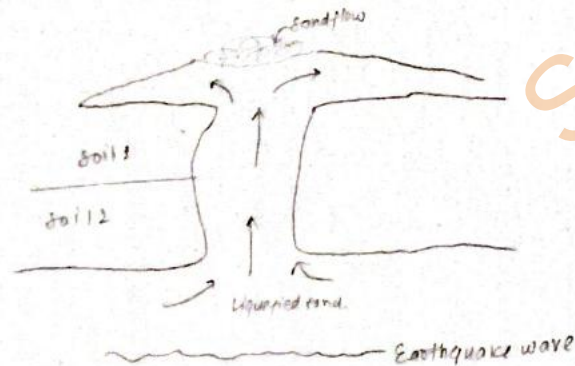
When, $u = \sigma$

$[\sigma' = 0] \leftarrow$ Liquefaction.

↳ Before earthquake:
 Soil grains in contact, pore water filling voids, effective stress present.

• During earthquake:
 Cyclic shaking causes grain rearrangement, pore pressure increases.

• At liquefaction:
 Grains lose contact, soil flows like a fluid.



Causes of liquefaction

1. Loose saturated sandy/tilty soil
 ↳ low relative density ($D_r < 50\%$)
2. High ground water table
 ↳ soil must be saturated
3. Strong and prolonged shaking
 ↳ sufficient energy to build up pore pressure
4. Uniform grain size
 ↳ Poorly graded soil more susceptible
5. Young deposits
 ↳ recent alluvial deposits are more vulnerable
6. Shallow depth
 ↳ typically occurs within 15m of surface

↳ Kathmandu valley is underlain by lacustrine (lake) deposits and alluvial sediments, making it highly susceptible to liquefaction.

↳ Sand boils and ground settlement were observed in several areas during the 2015 Gorkha Earthquake.

Q. No. 2 (b).

Define seismic hazard analysis explain how PSHA is carried out with steps.

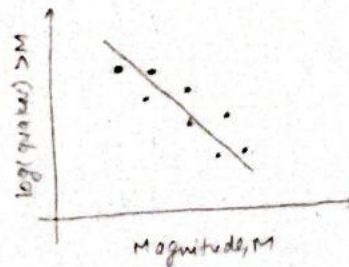
→ Seismic hazard analysis is the method of getting an estimate of the strong motion parameters at a site for the purpose of earthquake resistant design or seismic safety assessment.

→ Steps of PSHA:

(1) Identification and characterization of all earthquake source and probabilistically. This helps to identify source and sets how likely each point from source to site may have been affected.



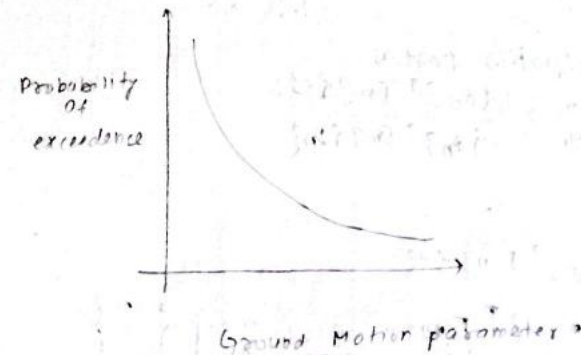
(2) Characterization of seismicity of each source zone which is done by a recurrence relationship indicating the average rate at which earthquake of a particular site will be affected.



(3) Determination of Motion from each source zones: In this step, the uncertainty inherent in predictive relationship (Attenuation law) is included in PSHA analysis.



(4) Finally, probabilistic calculations are done to combine uncertainty in earthquake location, earthquake size and ground to finally obtain the probability of that the ground motion parameter will be exceeded during a particular time period.



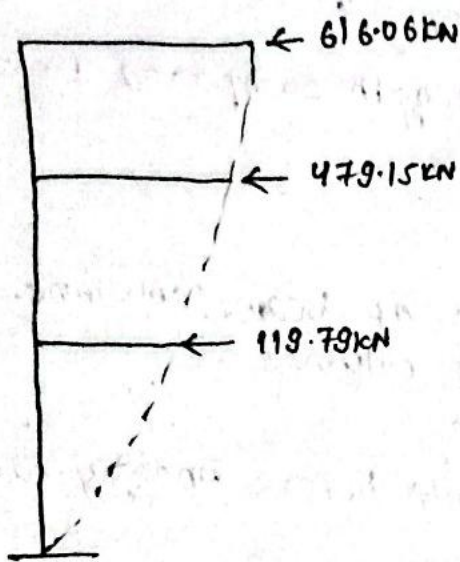


Fig: Lateral force distribution displacement

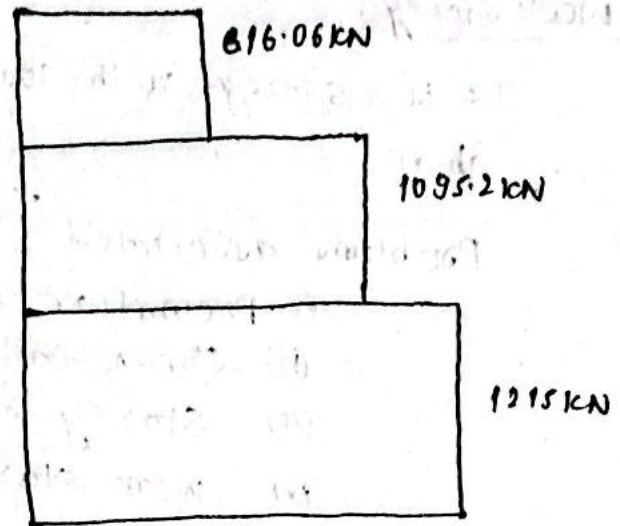


Fig: Storey shear distribution.

[Q. 5(b)]

Explain the problems associated with soft and weak stories in buildings.

Soft storey

It is a storey with low stiffness compared to storeys above.

Problems associated:

- (i) Excessive interstorey drift
- (ii) Damage concentration
- (iii) P- Δ (P-Delta) effects
- (iv) Non-structural damage like failure of infill walls, glazing, etc.
- (v) Columns form plastic hinges at same level and can lead to collapse mechanism.
- (vi) Increase in torsional response.

Weak storey

It is a storey with lower strength compared to storey above.

Problems associated

- (i) Premature failure of beam/columns.
- (ii) Shear failure of columns
- (iii) Storey collapse
- (iv) weak storey fails before energy can be distributed.
- (v) Progressive failure i.e. failure at one level leads to difficulty in load transfer causing chain failure.
- (vi) Load can't be safely transferred to foundation.

(6)

(c) Modal mass and Modal participation Factor

→ Modal mass is a measure that classifies the importance of mode shape when a structure is excited by base acceleration. It is denoted by

$\frac{L_n^2}{M_n}$ and given as:

$$\frac{L_n^2}{M_n} = \frac{\left[\sum_{i=1}^n \phi_{in} m_i \right]^2}{\sum_{i=1}^n \phi_{in}^2 m_i}$$

It is the portion of total structural mass that participates in a given vibration mode during dynamic analysis. Modes with higher effective modal mass contribute more to response.

Mode participation factor is defined as a measure of how strongly given mode contributes to the response of structure when subjected to force or displacement excitation in specific direction.

It is denoted by ' λ_n ' and given by:

$$\lambda_n = \frac{L_n}{M_n} = \frac{\{\phi_n\}^T [m] \{1\}}{\{\phi_n\}^T [m] \{\phi_n\}}$$

$$\lambda_n = \frac{\sum_{i=1}^n \phi_{in} m_i}{\sum_{i=1}^n \phi_{in}^2 m_i}$$

(d) Ductility and Energy absorption capacity in structure,
→ Ductility is defined as the ability of material to undergo large deformations without rupture before failure.
Ductility in concrete is defined by percentage of steel reinforcement within it.
Mild steel is an example of ductile material that can be bent and twisted without rupture.

→ Mathematically,

$$\text{Ductility} = \frac{\text{Maximum deformation}}{\text{Yield deformation}}$$

→ Ductility behaviour allows structure to undergo large plastic deformation with little decrease in strength.

Factors affecting ductility

- (i) Tension steel ratio
- (ii) Compression steel ratio
- (iii) Shape of cross-section
- (iv) Lateral reinforcement
- (v) Characteristic strength of steel and concrete

→ Energy absorption capacity of a structure is the ability of a structure to absorb and dissipate energy imparted during earthquakes without collapsing. There are different types of energy associated in a building during earthquake:

- (i) Elastic strain energy
- (ii) Kinetic energy
- (iii) Damping energy
- (iv) Hysteretic energy

Increasing energy absorption capacity of building:

- (i) Make structure ductile
- (ii) Make multiple load paths
- (iii) Proper detailing of RCC structures
- (iv) Provide necessary damping

(i) Richter scale / local magnitude

$$M_L = \log_{10} A - \log_{10} A_0$$

$\rightarrow M_L = \log_{10} \left(\frac{A}{A_0} \right)$

$\rightarrow$ max = force amplitude in μm
($T = 0.8 \text{ sec}$, $\gamma = 0.8$, $A_f = 2800$)
exactly at 100 km

$\rightarrow$ amplitude for zero magnitude (μm)
for epicentral distance other than
100 km

(ii) Surface wave (M_S)

$$M_S = \log_{10} \left(\frac{A}{T} \right) + 1.66 \log_{10} \Delta + 3.30$$

$$M_S = \log_{10} A + 1.66 \log_{10} \Delta + 2$$

$A \rightarrow$ max ground amplitude in μm .
 $\Delta \rightarrow$ epicentral distance measured in degree
($360^\circ \equiv$ circumference of earth)

$T = 20 \text{ sec}$.

(iii) Body wave (M_b)

$$M_b = \log_{10} A - 1.66 \log_{10} T + 0.01 \Delta + 5.9$$

$A \rightarrow$ amplitude of P-wave in μm
 $T \rightarrow$ time period of P-wave ($\approx \text{sec}$)
 $\Delta \rightarrow$ distance of seismograph from the epicenter in $^\circ$

(iv) Moment Magnitude (M_w)

$$[M_0 = \mu \times A \times d]$$

$\rightarrow$ shear strength of rock
 $A \rightarrow$ Area of dislocation or fault surface.
 $d \rightarrow$ Avg. displacement or slip on the fault plane.
 $M_0 \rightarrow$ seismic moment.

$$[M_w = \frac{2}{3} \log_{10} M_0 - 6.03]$$

$M_0 \rightarrow$ in Nm (Newton-metre)

If M_0 in dyne-cm [$1 \text{ dyne-cm} = 10^{-7} \text{ Nm}$]

$$M_w = \frac{2}{3} [\log_{10} M_0 - 16]$$

Energy released in an EQ.

$$\rightarrow [\log_{10} E = 4.4 + 1.5 M_S]$$

$E \rightarrow$ Energy released (E) in Joules.

If E in ergs ($1 \text{ joule} = 10^7 \text{ ergs}$)

$$[\log_{10} E = 11.8 + 1.5 M_S]$$

Epical distance of the seismometer (Δ) = $\frac{L}{2\pi R} \times 360^\circ$

$L \rightarrow$ distance to
seismograph
from
epicenter

$$2\pi R \rightarrow 360^\circ$$

$$1 \rightarrow \frac{360^\circ}{2\pi R}$$

$$L \rightarrow \Delta = \frac{L}{2\pi R} \times 360^\circ$$

Internal Assessment

(2082 Chaitra)

Subject:- Earthquake Resistant Design of Structures (Elective II)

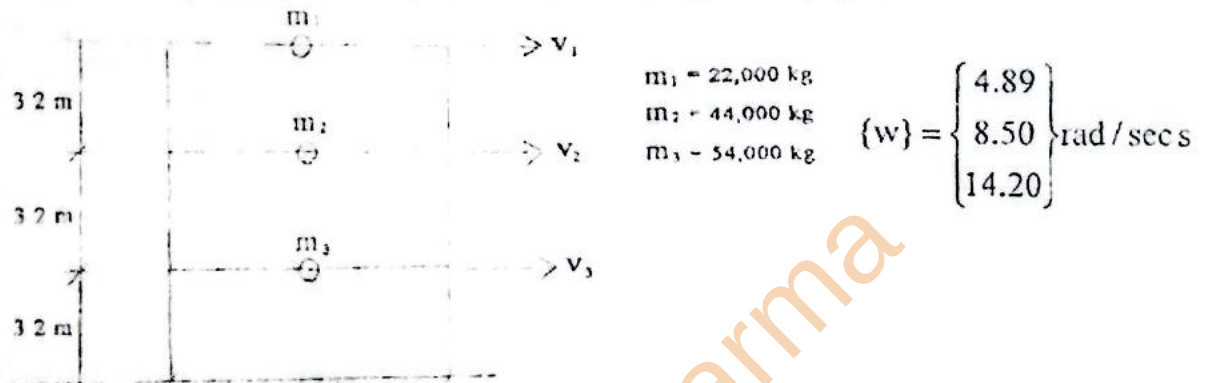
Attempt All questions.

The figures in the margin indicate **FULL MARKS**.

Duration: 2 hrs.

Assume Suitable data if necessary.

- 1 Estimate the moment magnitude of an event with rupture length of 150 km, rupture width 45 km and slip of average fault of 2.0 m. Take modulus of rigidity as $3.5 \times 10^{10} \text{ N/m}^2$ [10]
- 2 Using an appropriate attenuation law, estimate PGA at a site which is 250 km from MCT (3.3) with $a = 6.2$, $b = 0.8$ having 15% probability of an exceedance in 50 years [10]
- 3 A three storey building shown in figure below has the following vibration properties



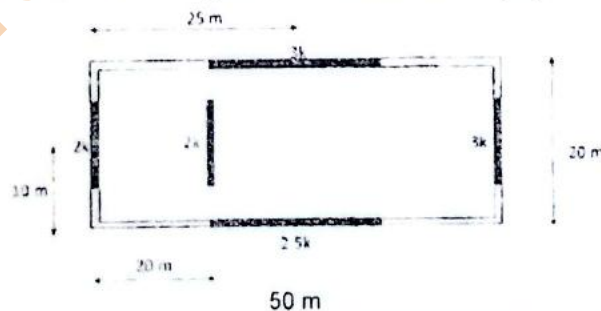
$$[\phi] = \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 0.51 & -1.50 & -6.24 \\ 0.20 & -0.87 & 14.12 \end{bmatrix}$$

and pseudo spectral velocity responses

$$\{S_v\} = \begin{Bmatrix} 0.65 \\ -0.24 \\ 0.37 \end{Bmatrix} \text{ m/sec}$$

Determine for each mode of vibration, the maximum displacement, lateral force, shear force and overturning moments at each storey level. Also determine total maximums due to all modes using SRSS method [30]

- 4 A one storey building of size 50m x 20m has walls of thickness 350 mm. The storey height is 4 m and the floor consists of cast in situ reinforced concrete. The building is subjected to lateral load of 500kN due to earthquake excitation in either directions. Calculate the design lateral force on the walls. [25]



- 5 For the three storey Education building frame, the seismic weights on the floor are: weight of first floor = 1800 kN, weight of second floor = 1500 kN, weight of third (top) floor = 650 kN. The building is located in Kalimati area.

The free vibration results are: natural periods $\{T_n\}^T = \{0.883, 0.404, 0.302\}$ sec and mode shapes

$\{\phi_1\}^T = \{0.25, 0.80, 1.00\}$, $\{\phi_2\}^T = \{-1.20, -0.10, 1.00\}$, and $\{\phi_3\}^T = \{0.25, -0.75, 1.00\}$. Determine the seismic force by dynamic analysis procedure as per NBC 105 [25]

(Best of Luck)

Lateral Load Resisting Systems for Building

① Moment Resisting Frames

↳ By flexure action &
rigid joint action

② Shear walls

↳ by shear phenomenon.

③ MRF

• Centre of mass: (C_m)
(Centroid)

↳ In which entire mass of the body may
be assumed to be concentrated.

∴ Mass is uniformly distributed : $C_m \equiv \text{centroid}(\bar{x})$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + \dots}{A_1 + A_2 + \dots} = \frac{\sum A_i x_i}{\sum A_i}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + \dots}{A_1 + A_2 + \dots} = \frac{\sum A_i y_i}{\sum A_i}$$

If mass is non uniformly distributed: $C_m \equiv \bar{x}$

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum W_i x_i}{\sum W_i} \quad (\text{In terms of mass or wt.})$$

$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\sum W_i y_i}{\sum W_i}$$

∴ Earthquake ^(F_{eq}) load passes through C_m .
as $[F_{eq} \propto \text{mass}]$

- centre of stiffness/rigidity
↳ stiffness concentrated pt. $\rightarrow C_s$ or C_p

EQ $\rightarrow$ passes $\rightarrow C_m$

EQ load acts result not from resultant pair $\rightarrow C_g \rightarrow C_g$

If, EQ load $\rightarrow$ passes $\rightarrow C_s$

Then, rigid body translation
& no rigid body rotation
(no moment generated)

Formula:

$$\bar{x}_{ce} = \frac{\sum K_i x_i}{\sum K_i} \quad [K \rightarrow \text{stiffness}]$$

$$\bar{y}_{ce} = \frac{\sum K_i y_i}{\sum K_i}$$

For $\bar{x}_{ce} \Rightarrow K_i \rightarrow$ only of frames aligned along y direction

|||

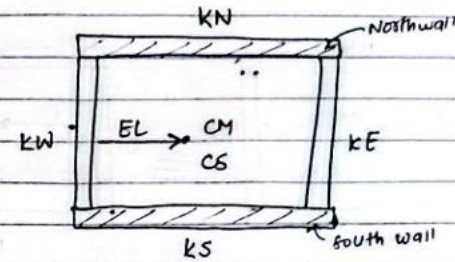
For $\bar{y}_{ce} \Rightarrow K_i \rightarrow$ " " " " " x direction

≡

① $C_m = C_s \rightarrow$ coincide

② $C_m \neq C_s \rightarrow$ don't coincide

① $C_m = C_s$ (torsionally uncoupled system)



~ rigid body translation
~ no rigid body rotation

Torsionally uncoupled system

(torsion of story couple system)

EL or $E\phi_x \rightarrow$ North wall & South wall of story
stiffness $\rightarrow$ acc. to divide/share
of E

Eg: For North wall: $L_N = \left[\frac{K_N}{K_N + K_S} \times E\phi_x \right] \leftarrow$ load shared by North wall (opposing/destabilizing force)

$K \uparrow \Rightarrow$ more load shared.

For south wall: $L_S = \left[\frac{K_S}{K_N + K_S} \times E\phi_x \right] (\leftarrow)$

Thus, $[L_N + L_S = E\phi_x]$

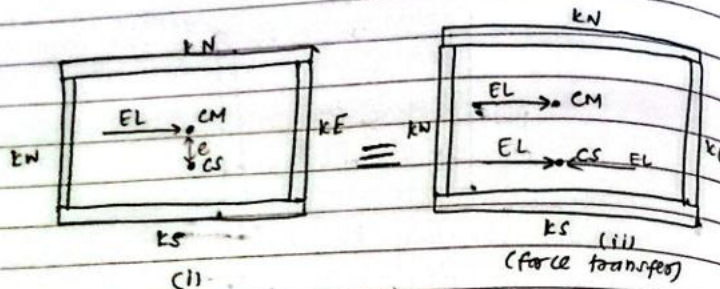
Direction: $E\phi_x \rightarrow$
(opposite to EQ load)

$L_N \& L_S \leftarrow$

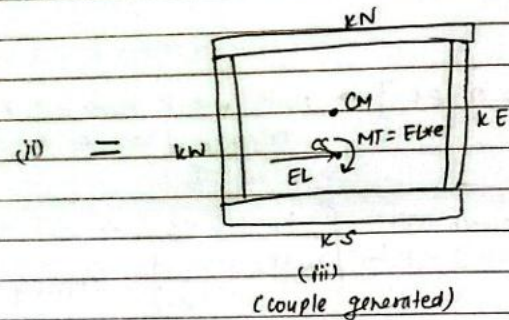
y-direction wall $\Rightarrow$ NO load shared.

Force = 'Force' + 'Couple'

⑧ CM $\neq$ CS (Torsionally coupled system)

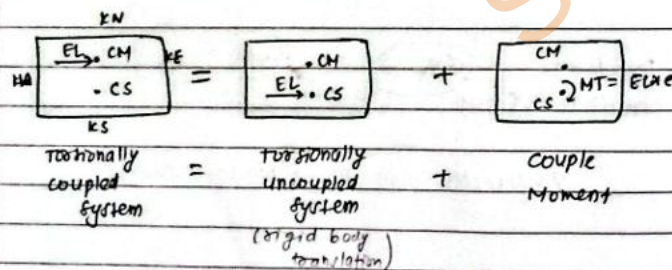


$e = e_y \rightarrow$ eccentricity along y direction.



couple $\rightarrow$ free vector (can keep anywhere)
but here kept at CS

Thus,



torsionally coupled = torsionally uncoupled + couple moment

$$[F_i = \frac{k_i}{\sum k_i} \times EL]$$

$$[F_i = \frac{k_i \sigma_i}{\sum k_i \sigma_i^2} \times M_T]$$

$$[M_T = \text{torsional moment} = EL \times e]$$

$F_i \rightarrow$ Force in i th wall

$k_i \rightarrow$ Stiffness of i th wall

$\sigma_i \rightarrow$ distance of frame ' i ' from CS

Thus:

Step 1: Determination of Centre of Mass (CM)

Step 2: " " " " Stiffness / Rigidity (CR)

Step 3: " " the type of system

Step 4: Calculation of Force in each frame due to eqk. load passing through CR (Torsionally uncoupled system)

Step 5: " " " " " " " " Torsional moment about CR (Couple Moment)

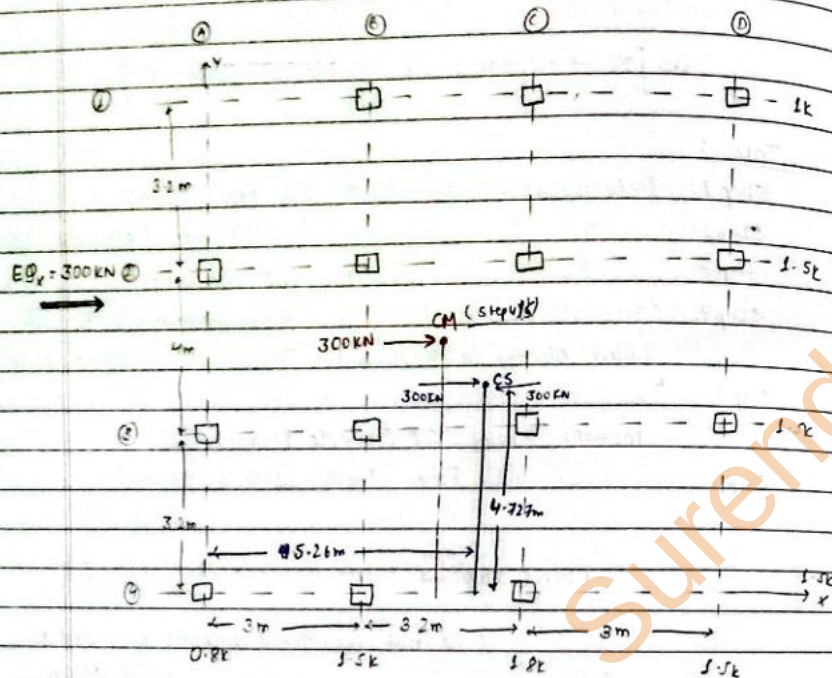
Step 6: " " Final Forces (sum of the two)

$\rightarrow$ seeing direction.

(Find ^{Design} additional eccentricity if needed) $\rightarrow$ Cl-7.8.2

(If not mentioned code) $\rightarrow$ no need to find design eccentricity

9. The figure given below shows the plan of an one storey building which could be considered as composed of 2D frames along the orthogonal directions. The roof diaphragm is rigid in its own plane and mass of the roof is uniformly distributed. The building is subjected to a lateral load of 300 kN due to earthquake load in X-direction and passing through the CM of the building. Calculate the lateral forces in the 2D-Frames.



Total frames: 4 in X-direction
4 in Y-direction

A stiffness of frame in each direction is given

Step 1: CM

Lumped mass system is applied

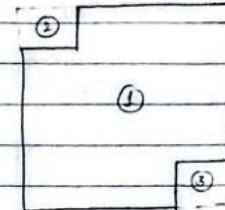
(mass of the entire story/floor is lumped at floor or roof)

We calculate overall mass at roof.

Mass uniformly distributed

CM = centroid

divide irregular geometry into regular shapes



$$\bar{X} = \frac{A_1 X_1 - A_2 X_2 - A_3 X_3}{A_1 - A_2 - A_3}$$

$$\bar{Y} = \frac{A_1 Y_1 - A_2 Y_2 - A_3 Y_3}{A_1 - A_2 - A_3}$$

$$A_1 = (9.2m) \times (10.4m) = 95.68m^2$$

$$X_1 = 9.2/2 = 4.6$$

$$Y_1 = 10.4/2 = 5.2$$

$$A_2 = 3 \times 3.2 = 9.6m^2$$

$$X_2 = 3/2 = 1.5$$

$$Y_2 = 10.4 - (3.2/2) = 9.2$$

$$A_3 = 3 \times 3.2 = 9.6m^2$$

$$X_3 = 9.2 - (3/2) = 7.7$$

$$Y_3 = 3.2/2 = 1.6$$

or

$$\bar{X} = \frac{(9.2 \times 10.4) \left(\frac{9.2}{2} \right) - (3 \times 3.2) \left(\frac{3}{2} \right) - (3 \times 3.2) \left(9.2 - \frac{3}{2} \right)}{9.2 \times 10.4 - 3 \times 3.2 - 3 \times 3.2} = 4.6m$$

$$9.2 \times 10.4 - 3 \times 3.2 - 3 \times 3.2$$

$$\bar{y} = \frac{(9.2 \times 10.4) \left(\frac{10.4}{2} \right) - (5 \times 5.2) \left(10.4 - 5 \frac{5.2}{2} \right) - (5 \times 5.2) \left(5 \frac{5.2}{2} \right)}{(9.2 \times 10.4) - (3 \times 5.2) - (3 \times 5.2)}$$

$$= 5.60 \text{ m}$$

Step 2: CS or CP

frame aligned along Y-direction
(A, B, C, D)

distance up to origin (here Y-axis)

$$\bar{x}_{cp} = \frac{\sum k_i x_i}{\sum k_i}$$

$$= \frac{0.8k \times 0 + (1.5k) \times 3 + (1.8k)(3+5.2) + (1.5k)(3+5.2+3)}{0.8k + 1.5k + 1.8k + 1.5k}$$

$$= 5.26 \text{ m}$$

frame aligned along X-direction
(1, 2, 3, 4)

(distance from X-axis)

$$\bar{y}_{cp} = \frac{\sum k_i y_i}{\sum k_i}$$

$$= \frac{1.5k \times 0 + 1.5k \times 3.2 + 1.5k \times (3.2+4) + 1k(3.2+4+3.2)}{1.5k + 1.5k + 1.5k + 1k}$$

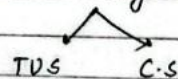
$$= 4.727 \text{ m}$$

Step 3: Type of System

Step 1: $\bar{x} = 4.6 \text{ m}$, $\bar{y} = 5.6 \text{ m}$ (CM)

Step 2: $\bar{x}_{cp} = 5.26 \text{ m}$, $\bar{y}_{cp} = 4.727 \text{ m}$ (CS)

The system is torsionally coupled system. (CM $\neq$ CS)



Step 4: Forces due to Earthquake load (TVS)

TVS

$$F_i = \frac{k_i}{\sum k_i} \times E Q_x \quad (\text{Required by frame } 1, 2, 3, 4)$$

(as EQ in X-direction)

$$F_1 = \frac{1k}{1k+1.5k \times 3} \times 300 = 54.55 \text{ kN} \quad (\leftarrow)$$

$$F_2 = \frac{1.5k}{1k+1.5k \times 3} \times 300 = F_3 = F_4 = 81.82 \text{ kN} \quad (\leftarrow)$$

$$F_3 = \frac{1.5k}{1k+1.5k \times 3} \times 300 = 81.82 \text{ kN} \quad (\leftarrow)$$

$$F_4 = \frac{1.5k}{1k+1.5k \times 3} \times 300 = 81.82 \text{ kN} \quad (\leftarrow)$$

Bub,

$$F_P = F_B = F_C = F_D = 0 \quad \text{NO force transferred}$$

(as frame has high inplane stiffness
low out-of-plane stiffness)

Step 5: Forces due to couple moments
(due to twisting moments)

$$F_i = \frac{k_i r_i}{\sum k_i r_i^2} \times M_T$$

$$M_T = E Q_x e$$

$$r_i = ? \quad (\text{distance of } i^{\text{th}} \text{ frame from CS})$$

$$\odot (e_x = \bar{x}_{cp} - \bar{x})$$

$$\text{As, } E Q_x, \Rightarrow e = e_y = |\bar{y}_{cp} - \bar{y}|$$

$$= |4.727 - 5.6|$$

$$e_y = 0.873 \text{ m}$$

From IS 1893, clause 7.8.2
If, $e_y > 0.05b$ } or, $e_x > 0.05l$

$$0.05b = 0.05 \times 10.4 \\ 0.5m$$

$$e_y = 0.873m > 0.5m$$

∴, need to calculate design eccentricity (e_{dy})

$$e_{dy} = 1.5e_y + 0.05b \\ = 1.5 \times 0.873 + 0.05 \times 0.5$$

$$[e_{dy} = 1.23m]$$

$$M_T = E_{dx} \times e_{dy} = 300 \times 1.23 = 369 kNm (\text{∩})$$

(CLW couple)

→ CM

→ ←

(∩) 1, 3, 4 → HZ frame

$$\begin{aligned} r_1 &= 10.4 - 4.727 = 5.673m \\ r_2 &= (3.2 + 4) - 4.727 = 2.473m \\ r_3 &= 4.727 - 3.2 = 1.527m \\ r_4 &= 4.727m \end{aligned}$$

A, B, C, D → VT frame

$$\begin{aligned} r_A &= 5.26m \\ r_B &= 5.26 - 3 = 2.26m \\ r_C &= (3 + 5.2) - 5.26 = 0.94m \\ r_D &= (3 + 3.2 + 3) - 5.26 = 3.94m \end{aligned}$$

$$F_i = \frac{E_i x_i}{\sum E_i x_i^2} \times M_T$$

$$[M_T = 369 kNm]$$

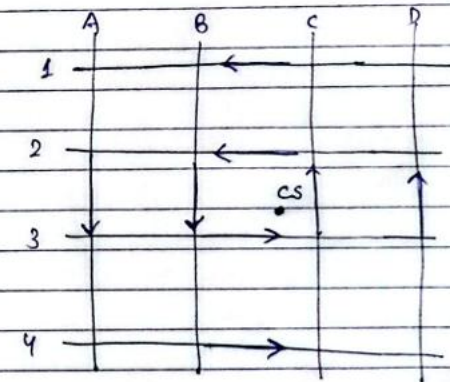
Frame	K_i	x_i	$K_i x_i$	$K_i x_i^2$	F_i (kN)
1	1k	5.673	5.673k	32.182k	15.75 (←)
2	1.5k	2.473	3.709k	9.173k	10.3 (←)
3	1.5k	1.527	2.29k	3.492k	6.35 (→)
4	1.5k	4.727	7.08k	33.4176k	19.65 (→)
A	0.8k	5.26	4.208k	22.134k	11.68 (↓)
B	1.5k	2.26	3.39k	7.66k	9.41 (↓)
C	1.8k	0.94	1.692k	1.59k	4.7 (↑)
D	1.5k	3.94	5.91k	23.28k	16.4 (↑)
$\sum K_i x_i^2$					132.929k

Step 6: Final forces (with direction)

For coupled force (see couple & identify)

For uncoupled → opposite to E_{dx} i.e. ←

369 kN (total resisting couple)
So to resist
A CLW couple to resist the acting couple (in each frame)



(force direction for creating resisting couple)

	TUS	C.S.	Total/Final force
F_1	←	←	$15.75(←) + 15.75(←) = 70.3(←)$
F_2	←	←	$81.82(←) + 10.3(←) = 92.12(←)$
F_3	←	→	$81.82(←) + 6.35(→) = 75.47(←)$
F_4	←	→	$81.82(←) + 19.65(→) = 62.17(←)$
F_5	0	↓	$0 + 11.68(↓) = 11.68(↓)$
F_6	0	↓	$0 + 9.41(↓) = 9.41(↓)$
F_7	0	↓	$0 + 4.70(↓) = 4.70(↓)$
F_8	0	↑	$0 + 16.4(↑) = 16.4(↑)$
F_9	0	↑	

MDOF System

↳ Free undamped vibration

Equation of motion for MDOF:

$$m\ddot{x} + c\dot{x} + kx = 0 \rightarrow \text{SDOF}$$

undamped, free vibration

$$[m\ddot{x} + kx = 0]$$

For MDOF

$$[M]\ddot{x} + [K]x = 0 \leftarrow \text{MDOF}$$

vibration: simple harmonic motion

$$[K]x - \omega^2[M]x = \{0\} \quad \text{--- (1)}$$

$$[A]x = \{0\}$$

↑ Homogeneous equation

Two solution

Trivial solution

$$\{x\} = 0$$

(NOT taken)

Non-trivial sol's

$$[A] = 0$$

$$|A| = 0$$

For Non-trivial solution:

$$[K] - \omega^2[M] = 0 \quad \text{--- (2)}$$

Say, $[K]_{n \times n} \rightarrow$ Degree of Freedom.

$$[M]_{n \times n}$$

Then, n nos. of ω

$\omega \rightarrow$ frequency

least $\omega \rightarrow$ normal frequency / natural frequency

$n = \text{DOF} = \text{deg. no. of SDOF}$

Say, 3 story building

$$\begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} \rightarrow \text{frequency vector}$$

Now, For ω_1 , find $\{X_{01}\} = \{ \}$ ← relative displacement vector

$$\omega_2, \{X_{02}\} = \{ \} \quad \text{Node shape } \{\Phi\}$$

$$\omega_3, \{X_{03}\} = \{ \}$$

But, Rank < Order $\rightarrow$ infinite no. of solutions
(to we don't get exact displacement)

either ~~we~~ it is normalized
to obtain relative displacement
(by considering one of the displacements)

MDOF System

1. Generalized normal coordinate mass

$$M_n = \{\Phi_n\}^T [M] \{\Phi_n\}$$

2. Earthquake excitation factor

$$L_n = \{\Phi_n\}^T [M] \{1\}$$

Unit matrix (column unit matrix)

3. Mode Participation factor

$$\frac{L_n}{M_n} = \frac{\{\Phi_n\}^T [M] \{1\}}{\{\Phi_n\}^T [M] \{\Phi_n\}}$$

4. Effective modal mass

$$\frac{L_n^2}{M_n} = \frac{[\{\Phi_n\}^T [M] \{1\}]^2}{\{\Phi_n\}^T [M] \{\Phi_n\}}$$

5. Modal maximum displacement

$$\{V_{n,max}\} = \{\Phi_n\} \frac{L_n}{M_n} S_d(\xi_n, T_n) \quad \text{IS 1893}$$

For MDOF

Say, 3 floor

↓
3 node

↓
3 values of
max displacement

S_d = spectral displacement

$$\& S_v = \omega S_d$$

$$S_a = \omega S_v = \omega^2 S_d = \text{spectral acceleration}$$

$$\therefore \{V_{n,max}\} = \{\Phi_n\} \frac{L_n}{M_n} \frac{S_a(\xi_n, T_n)}{\omega_n^2}$$

6. Modal maximum force

Stiffness matrix

$$\{F = k \times a\} \quad \text{elastic force}$$

$$\{F_{n,max}\} = [K] \{V_{n,max}\}$$

$$= [K] \{\Phi_n\} \frac{L_n}{M_n} \frac{S_a(\xi_n, T_n)}{\omega_n^2}$$

$$= \omega_n^2 [M] \{\Phi_n\} \frac{L_n}{M_n} \frac{S_a(\xi_n, T_n)}{\omega_n^2}$$

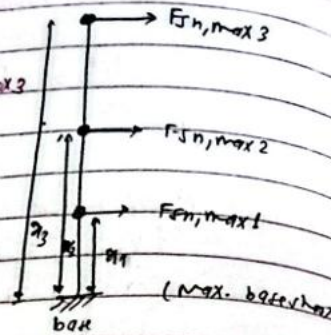
$$= [M] \{\Phi_n\} \frac{L_n}{M_n} S_a(\xi_n, T_n)$$

$$\{F_{n,max}\} = \begin{Bmatrix} F_{n,max1} \\ F_{n,max2} \\ F_{n,max3} \end{Bmatrix}$$

7. Base Shear (Max).

$$\text{Max base shear} = F_{n,max1} + F_{n,max2} + F_{n,max3}$$

'OR'



$$\{V_{n,max}\} = \{1\}^T \{F_{n,max}\}$$

$$\{1\} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$= \{1\}^T [M] \{\phi_n\} \frac{L_n \sin(\xi_n, T_n)}{M_n}$$

$$= \frac{L_n \cdot L_n}{M_n} \cdot \sin(\xi_n, T_n)$$

$$= \frac{L_n^2}{M_n} \cdot \sin(\xi_n, T_n)$$

↑ Spectral acceleration
↑ effective modal mass

8. Overturning moment of each mode of vibration.

$$M_0 = \{X\}_{1 \times 3} \{F_{n,max}\}_{3 \times 1}$$

$$= \{X\} \frac{L_n}{M_n} [M] \{\phi_n\} \sin(\xi_n, T_n)$$

$$M_0 = (F_{n,max1}) * X_1 + (F_{n,max2}) * X_2 + (F_{n,max3}) * X_3$$

$$M_0 = \{X_1 \ X_2 \ X_3\} * \begin{Bmatrix} F_{n,max1} \\ F_{n,max2} \\ F_{n,max3} \end{Bmatrix}$$

$$L_n = \{\phi_n\}^T [M] \{1\}$$

'OR'

$$\{1\}^T [M] \{\phi_n\}$$

Maximum response due to effect of all modes.

$$\left[\begin{array}{l} \text{Diff. value} \\ \text{of } w \\ \text{diff mode} \end{array} \right] \rightarrow \left\{ \begin{array}{l} \text{Max. value of} \\ \text{node force, max base shear force, overturning} \\ \text{moment} \end{array} \right\}$$

say, V_{B1}, V_{B2}, V_{B3}
max. in each mode

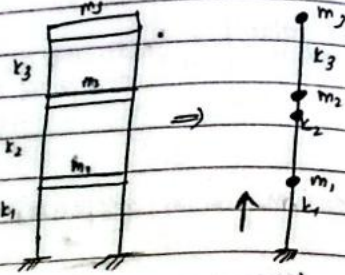
For Maximum response $\Rightarrow$ SRSS method

(Maximum of all modes)

SRSS = Square root of sum of squares method.

$$V_B = \sqrt{V_{B1}^2 + V_{B2}^2 + V_{B3}^2}$$

How to write mass matrix & stiffness matrix?



$[M]$ = Mass matrix: $n \times n$

$[K]$ = Stiffness matrix: $n \times n$

$n \rightarrow$ DOF / no. of story

Here,

$$[n = 3]$$

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}_{3 \times 3} \quad (\text{lumped mass matrix})$$

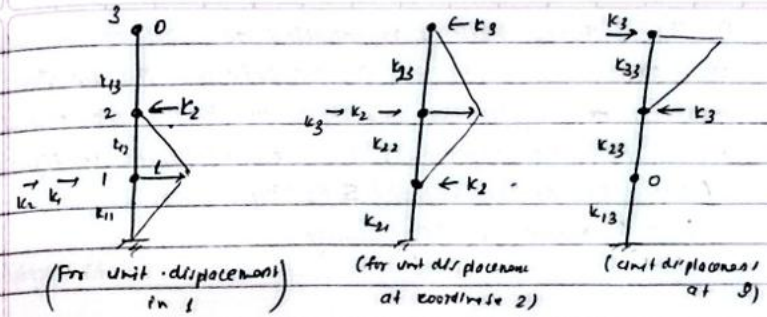
$[M] \rightarrow$ two ways $\rightarrow$ (i) column mass neglect (lumped mass)
(ii) column mass add stiffness at floor 2
at floor 2 in lumped mass

$$[K] = [k_{ij}] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}_{3 \times 3}$$

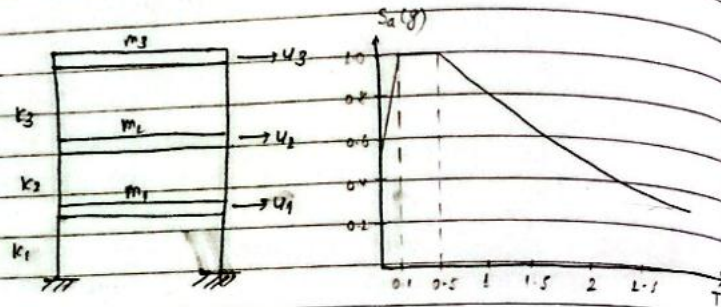
i = force at coordinate 'i'

j = displacement at coordinate 'j'

$$= \begin{bmatrix} (k_1 + k_2) & -k_2 & 0 \\ -k_2 & (k_2 + k_3) & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}_{3 \times 3}$$



Q. A three storey building is modeled as 3-DOF system with rigid floor as shown in the figure. Determine the top floor maximum displacement and shear due to the eqk. ground motion using the response spectrum method (given in figure). Assume $K_3 = K_2 = 3K_1$ & $m_3 = 1.5 m_2 = m_1$, where $K_1 = 2000 \text{ kN/m}$ & $m_1 = 2500 \text{ kg}$ [2073 Magh]



→ Solⁿ:

$$\left. \begin{aligned} m_1 &= 2500 \text{ kg} \\ m_2 &= \frac{m_1}{1.5} = 1666.67 \text{ kg} \\ m_3 &= m_1 = 2500 \text{ kg} \end{aligned} \right\} \begin{aligned} k_1 &= 2000 \text{ kN/m} \\ k_2 &= 3k_1 = 6000 \text{ kN/m} \\ k_3 &= k_2 = 6000 \text{ kN/m} \end{aligned}$$

$$\uparrow \text{Mass matrix } [M] = \begin{bmatrix} 2500 & 0 & 0 \\ 0 & 1666.67 & 0 \\ 0 & 0 & 2500 \end{bmatrix} \quad \text{Eq} = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 1.67 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \times 10^3$$

$$\text{Stiffness matrix } [K] = \begin{bmatrix} (k_1+k_2) & -k_2 & 0 \\ -k_2 & (k_2+k_3) & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} = \begin{bmatrix} 8000 & -6000 & 0 \\ -6000 & 12000 & -6000 \\ 0 & 0 & 6000 \end{bmatrix} \text{ kN/m}$$

$$[K] = \begin{bmatrix} 8 & -6 & 0 \\ -6 & 12 & -6 \\ 0 & 0 & 6 \end{bmatrix} \times 10^5 \text{ N/m}$$

≡ Eqⁿ of motion for undamped free vibration:

$$[K] - \omega^2 [M] = 0$$

$$\text{On } \begin{vmatrix} 8 & -6 & 0 \\ -6 & 12 & -6 \\ 0 & 0 & 6 \end{vmatrix} \times 10^5 - \omega^2 \begin{vmatrix} 2.5 & 0 & 0 \\ 0 & 1.67 & 0 \\ 0 & 0 & 2.5 \end{vmatrix} \times 10^3 = 0$$

$$\text{Say, } \omega^2/100 = B$$

$$\text{On } \begin{vmatrix} (8-2.5B) & -6 & 0 \\ -6 & (12-1.67B) & -6 \\ 0 & -6 & (6-2.5B) \end{vmatrix} = 0$$

B → cubic eqⁿ → 3 values

On solving

$$B_1 = 0.25 \rightarrow \omega_1 = 5 \text{ rad/s}$$

$$B_2 = 0.283 \rightarrow \omega_2 = 16.8 \text{ rad/s}$$

$$B_3 = 9.70 \rightarrow \omega_3 = 31.19 \text{ rad/s}$$

$$\begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} = \begin{Bmatrix} 5 \\ 16.8 \\ 31.19 \end{Bmatrix} \text{ rad/s} \leftarrow \text{Frequency vectors (3 modes)}$$

$$\text{as, } T = 2\pi/\omega$$

$$\text{Time period } \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 1.25 \\ 0.37 \\ 0.20 \end{Bmatrix} \text{ Sec}$$

3 Mode shape $\{\phi\}$
 $[K] - \omega^2 [M] \{\phi\} = \{0\}$

$$\begin{bmatrix} (8-2.58) & -6 & 0 \\ -6 & (12-1.678) & -6 \\ 0 & -6 & (6-2.58) \end{bmatrix} \begin{Bmatrix} \phi_{1n} \\ \phi_{2n} \\ \phi_{3n} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

First mode, (for ω_1 or B_1)

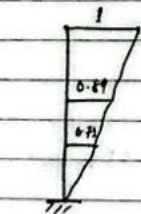
$$\{\phi_1\} = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \\ \phi_{31} \end{Bmatrix} = \begin{Bmatrix} 0.72 \\ 0.89 \\ 1 \end{Bmatrix}$$

2nd mode (B_2)

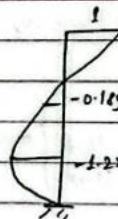
$$\{\phi_2\} = \begin{Bmatrix} \phi_{12} \\ \phi_{22} \\ \phi_{32} \end{Bmatrix} = \begin{Bmatrix} -1.22 \\ -0.189 \\ 1 \end{Bmatrix}$$

3rd mode (B_3)

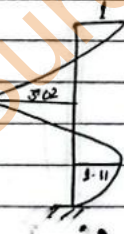
$$\{\phi_3\} = \begin{Bmatrix} \phi_{13} \\ \phi_{23} \\ \phi_{33} \end{Bmatrix} = \begin{Bmatrix} 1.11 \\ -3.02 \\ 1 \end{Bmatrix}$$



(Mode shape)



(2nd mode)



(3rd mode)

3 mode shapes

4 Generalized coordinate mass:

$$M_n = \{\phi_n\}^T [M] \{\phi_n\}$$

For 1st mode

$$M_1 = \{\phi_1\}^T [M] \{\phi_1\} = \begin{Bmatrix} 0.72 \\ 0.89 \\ 1 \end{Bmatrix}^T \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 1.67 & 0 \\ 0 & 0 & 2.5 \end{bmatrix} \times 10^3 \begin{Bmatrix} 0.72 \\ 0.89 \\ 1 \end{Bmatrix} = 5.112 \times 10^3 \text{ kg}$$

Equivalent SDOF mass for given MDOF system in mode 1

For 2nd mode

$$M_2 = \{\phi_2\}^T [M] \{\phi_2\} = 6.162 \times 10^3 \text{ kg}$$

For 3rd mode

$$M_3 = \{\phi_3\}^T [M] \{\phi_3\} = 20.80 \times 10^3 \text{ kg}$$

5. Earthquake excitation factor

$$\bar{L}_n = \{\phi_n\}^T [M] \{1\}$$

Mode 1:-

$$\bar{L}_1 = \{\phi_1\}^T [M] \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = 5.78 \times 10^3$$

Mode 2:-

$$\bar{L}_2 = \{\phi_2\}^T [M] \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = -0.234 \times 10^3$$

Mode 3:-

$$\bar{L}_3 = 0.231 \times 10^3$$

As asked to determine $\rightarrow$ Find $\left\{ \begin{array}{l} \text{① 3 max displacement (for 3 modes)} \\ \text{② 3 max base shear (for 3 modes)} \end{array} \right.$

$S_d \rightarrow \text{max value}$

$S_d(S_n, T_n)$

But given response spectrum ($S_d \rightarrow f^n(T_n)$)

$$T_1 = 1.25 \text{ s} \rightarrow S_{d,1} = 0.6g = 5.89 \text{ m/s}^2$$

$$T_2 = 0.375 \text{ s} \rightarrow S_{d,2} = 1g = 9.81 \text{ m/s}^2$$

$$T_3 = 0.18 \text{ s} \rightarrow S_{d,3} = 1g = 9.81 \text{ m/s}^2$$

6 Max. displacement

$$\{V_{n, \max}\} = \{ \phi_n \} \frac{L_n}{M_n} \cdot \frac{S_d}{\omega_n^2}$$

For mode 1: $\omega_1, L_1, M_1, S_{d,1}, \phi_1$

$$\{V_{1, \max}\} = \begin{Bmatrix} 0.019 \\ 0.024 \\ 0.027 \end{Bmatrix} \text{ m} \quad \begin{array}{l} \text{(check if } g \text{ is} \\ \text{considered or not)} \end{array}$$

Max. displacement of each node for mode 1

$$\text{Mode 1: } \{V_{1, \max}\} = \begin{Bmatrix} 0.00016 \\ 0.00024 \\ -0.00013 \end{Bmatrix} \text{ m}$$

$$\text{Mode 3: } \{V_{3, \max}\} = \begin{Bmatrix} -0.000001264 \\ -0.0000344 \\ -0.0000114 \end{Bmatrix} \text{ m} \quad \leftarrow \text{Top floor}$$

Now,

Max displacement of top floor

(using SRSS method)

$$= \sqrt{(0.027)^2 + (-0.00013)^2 + (-0.0000114)^2}$$

$$= 0.027 \text{ m}$$

7 Max. Base shear

$$V_{Bn, \max} = \frac{L_n^2}{M_n} \times S_{d,n}$$

$$\text{Mode 1: } V_{B1, \max} = \frac{L_1^2}{M_1} \times S_{d,1} =$$

$$\text{Mode 2: } V_{B2, \max} = \frac{L_2^2}{M_2} \times S_{d,2} =$$

$$\text{Mode 3: } V_{B3, \max} = \frac{L_3^2}{M_3} \times S_{d,3} =$$

$$\therefore [V_{B, \max} = 3.920 \text{ N}]$$

9) CDC method.

maxe modal response

$$R_1 = 100$$

$$R_2 = 80$$

$$R_3 = 50$$

Natural circular frequencies,

$$\omega_1 = 10 \text{ rad/s}, \omega_2 = 11 \text{ rad/s}, \omega_3 = 18 \text{ rad/s}$$

$$\xi = 5\% = 0.05$$

① Frequency ratio, $\beta_{12} = \frac{\omega_2}{\omega_1} = \frac{11}{10} = 1.10$

$$\beta_{13} = \frac{\omega_3}{\omega_1} = \frac{18}{10} = 1.80$$

$$\beta_{23} = \frac{\omega_3}{\omega_2} = \frac{18}{11} = 1.64$$

② Correlation coeff:

$$S_{ij} = \frac{8\xi^2(1+\beta)\beta^{1.5}}{(1-\beta^2)^2 + 4\xi^2\beta(1+\beta)^2}$$

③ S/W Mode 1 & 2 (closely spaced)

$$\beta_{12} = 1.10$$

$$S_{12} = 0.523$$

④ S/W Mode 1 & 3 (far apart)

$$\beta_{13} = 1.80$$

$$S_{13} = 0.026$$

⑤ S/W 2 & 3 (Moderately apart)

$$\beta_{23} = 1.64$$

$$S_{23} = 0.037$$

$$A_{10}, S_{11} = S_{22} = S_{33} = 1.$$

③ CDC for 3 mode, $R = \sqrt{\sum_{i=1}^3 \sum_{j=1}^3 S_{ij} R_i R_j}$

$$R = \sqrt{\frac{R_1^2}{100} + \frac{R_2^2}{80} + \frac{R_3^2}{50} + 2S_{12}R_1R_2 + 2S_{13}R_1R_3 + 2S_{23}R_2R_3}$$

$\frac{100}{100} \quad \frac{80}{80} \quad \frac{50}{50} \quad 0.523 \quad 0.026 \quad 0.037$

$$[R_{CDC} = 166.805]$$

Equivalent Lateral load Procedure (Seismic coefficient Method)

↳ Simplified method for EPDs

↳ At first base shear is calculated and then distributed along the ht. of the building

• Computation of Design seismic base shear

Clause 3.2.9 IS-1893

[CI-7.2.1-1893 (part 1): 2016]

Total design lateral force or design seismic base shear

$$[V_B = A_h \cdot W]$$

↑
Art. 6.4.2

W ← as per 7.4

(7.4) $\underline{W}$ (Seismic wt.)

Full dead load + appropriate amt. of Imposed load (7.3)

imposed load	% of IL
upto 3 kN/m ²	→ 25%
> 3 kN/m ²	→ 50%

6.4.2

A_h (design hz. seismic coeff)

$$A_h = \left(\frac{Z}{2} \right) \left(\frac{S_a}{g} \right)$$

$$\left(\frac{R}{I} \right)$$

Z → Zone factor (Table 3)

(or NBC)

I → Importance factor

R → Response reduction factor

(S_a/g) → design acc = coeff

$$\rightarrow \frac{1}{1} \rightarrow 7.6.2$$

(S_a/g) → Fig. 2

A 6.4.2

(for other damping 2%)

fundamental time period
7.6.2

② gen. zone → for ktm

$$\rightarrow 0.36$$

- $\frac{S_a}{g} \left(\frac{R}{I} \right), I, Z, I \rightarrow SS$
- Z = Zone factor for the maximum considered earthquake (MCE) and service life of structure in a zone.

The factor 2 in denominator of Z is used to reduce the zone factor for MCE to zone factor for Design Basic Earthquake (DBE)

- I = Importance factor which depends upon the functional uses of the structures, characterized by hazardous consequences of its failure, post earthquake functional needs, historical values, or economic importance.

- R = Response reduction factor which depends upon the perceived seismic damage performance of the structure, characterized by ductile or brittle deformations.

It should be noted that the ratio I/R should not be greater than 1.0

R has lower values for brittle structures and higher values for more ductile structures.

- S_a/g = Average response acceleration coefficient for rock or soil sites. It is based upon appropriate natural periods and damping of the structures.

#1 Approximate Analysis methods of frames

- A. Portal frame method
- B. Cantilever method

↳ by certain assumptions

↳ to solve frames subjected to lateral loads only

- ↳ wind load
- ↳ seismic / Ege load.

↳ gives approximate result

A. Portal frame method

Assumptions:

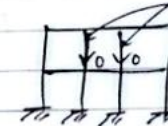
(1) Pt. of contraflexure (i.e. pt. of zero BM) in the column, is at the mid-ht of the columns

10 " "

" " " mid-span " " beams

beams

ii) Axial force in the internal columns is zero



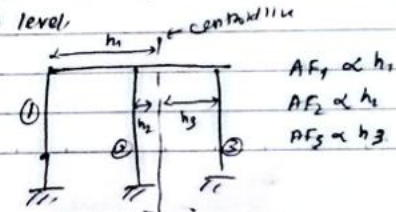
B. Cantilever method

Assumption:

c) is

(f) ✓

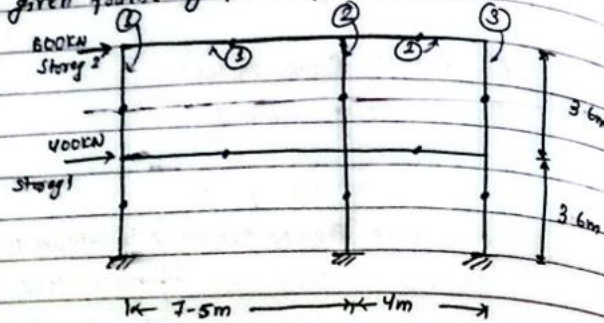
(iii) Axial force in the column at any floor is linearly proportional to its distance from the centroid of all the columns at that level.



Q. Analyze the given frame by portal frame method.

AFD, SFD, BMD

• Naming



Solⁿ:

• → pt. of contraflexure

Step 1: cut section in upper storey (story 2)



Considering eqⁿ of upper storey solve the forces (AF)

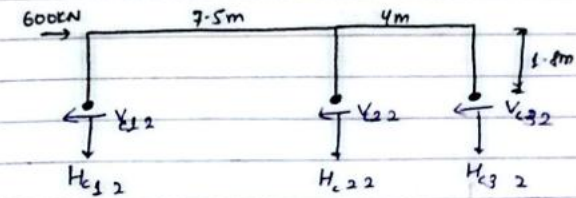
we get AF for all 3 columns.

Step 2: cut section in first storey
(cut section passes through mid)



• solve eqⁿ condition to find AF in lower columns

Step 1:

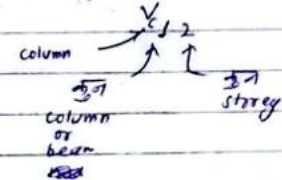


⊙ Assume direction of H & V at section

For column: H → axial force

V → shear force

(Tension)



⊙ $[H_{22} = 0]$ ← Assumption (iii)

⊙ For left most column

(at pt. of contraflexure)

$$+ \sum M = 0$$

$$\text{on } 600 \times 3.6 + H_{32} \times (7.5 + 4) = 0$$

$$\Rightarrow H_{32} = -93.91 \text{ kN}$$

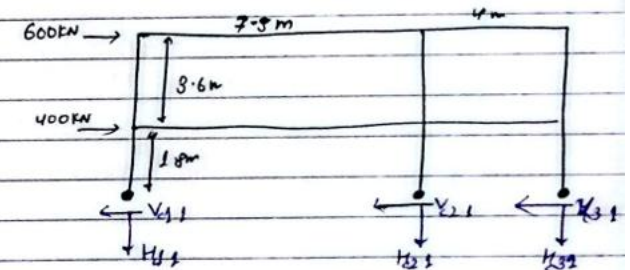
$$[H_{32} = 93.91 \text{ kN (C)}]$$

⊙ $+ \sum F_y = 0$

$$-H_{12} - H_{32} = 0$$

$$\Rightarrow [H_{12} = 93.91 \text{ kN (T)}]$$

Step 2:



$$\odot (M_{B1} = 0)$$

At leftmost pt:

$$+ \uparrow \Sigma M = 0$$

$$\text{or } 600 \times (3.6 + 1.8) + 400 \times (1.8) + M_{B1} \times (7.5 + 4) = 0$$

$$M_{B1} = -344.348 \text{ kN}$$

$$[M_{B1} = 344.348 \text{ kN (C)}]$$

$$\odot + \uparrow \Sigma F_y = 0$$

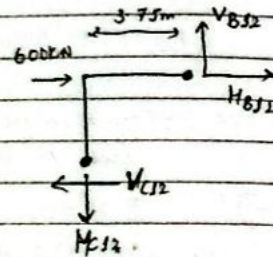
$$\text{or } -H_{B1} - H_{C1} = 0$$

$$\text{or } [H_{C1} = 344.348 \text{ kN (T)}]$$

Next,

consider

(leftmost to position)



Now for beam

$$(i) + \uparrow \Sigma M = 0 \text{ (at pt. of contraflexure of beam)}$$

$$\text{or } V_{C12} \times 2.8 - H_{C12} \times 3.75 = 0$$

$$\Rightarrow [V_{C12} = 195.652 \text{ kN}]$$

$$(ii) + \rightarrow \Sigma F_x = 0$$

$$\text{or } 600 + H_{B12} - V_{C12} = 0$$

$$\text{or } H_{B12} = 195.652 - 600$$

$$H_{B12} = -404.348 \text{ kN}$$

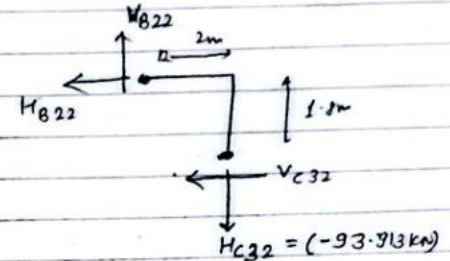
$$[H_{B12} = 404.348 \text{ kN (C)}]$$

$$(ii) + \uparrow \Sigma F_y = 0$$

$$\text{or } -H_{C12} + V_{B12} = 0$$

$$[V_{B12} = 93.913 \text{ kN}]$$

Next rightmost to top.



$$(i) + \uparrow \Sigma M = 0$$

$$\text{or } H_{C32} \times 2 + V_{C32} \times 1.8 = 0$$

$$\text{or } -93.913 \times 2 + V_{C32} \times 1.8 = 0$$

$$\Rightarrow [V_{C32} = 104.348 \text{ kN}]$$

$$(ii) + \rightarrow \Sigma F_x = 0$$

$$\text{or } -H_{B22} - V_{C32} = 0$$

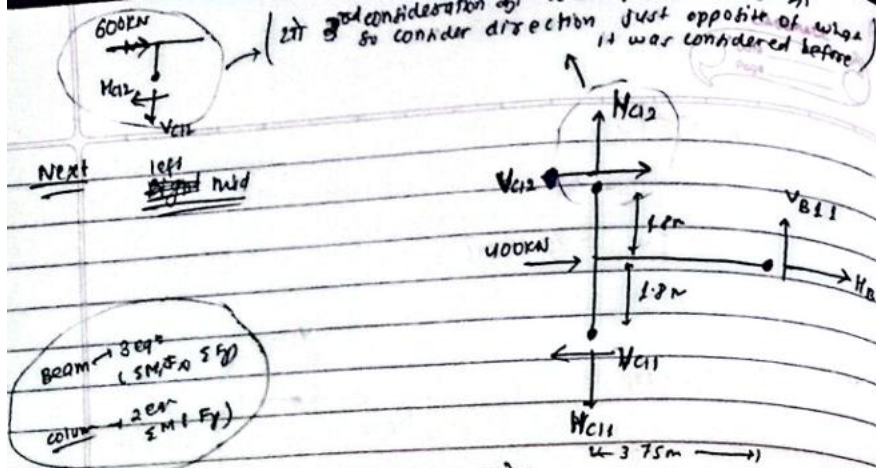
$$[H_{B22} = -104.348 \text{ kN}]$$

$$[H_{B22} = 104.348 \text{ kN (C)}]$$

$$(iii) + \uparrow \Sigma F_y = 0$$

$$\text{or } V_{B22} - H_{C32} = 0$$

$$[V_{B22} = 104.348 \text{ kN}]$$



(i) $\sum M = 0$ (beam at support)

on $H_{C2} \times \frac{1.8}{3.75} + V_{C2} \times \frac{1.8}{3.75} - H_{C1} \times \frac{3.75}{3.75} + V_{C1} \times \frac{1.8}{3.75} = 0$

$[V_{C1} = 325.788 \text{ kN}]$

(ii) $\sum F_x = 0$

on $V_{C2} + 400 - V_{C1} + H_{B1} = 0$

$H_{B1} = 270.164 \text{ kN}$

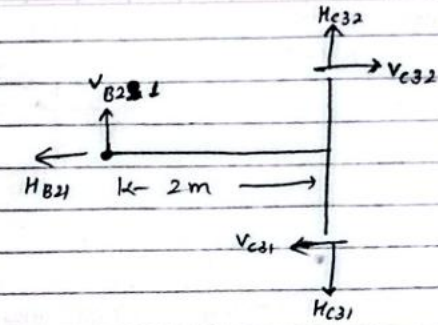
$[H_{B1} = 270.164 \text{ kN (C)}]$

(iii) $\sum F_y = 0$

on $H_{C2} - H_{C1} + V_{B1} = 0$

$[V_{B1} = 250.405 \text{ kN}]$

Next mid Right



(i) $\sum M = 0$

on $V_{C2} \times 1.8 + V_{C3} \times 1.8 + H_{C3} \times 2 - H_{C2} \times 2 = 0$

on $[V_{C3} = 178.913 \text{ kN}]$

(ii) $\sum F_x = 0$

on $V_{C2} + H_{B2} - V_{C3} = 0$

=

$H_{B2} = -69.565 \text{ kN}$

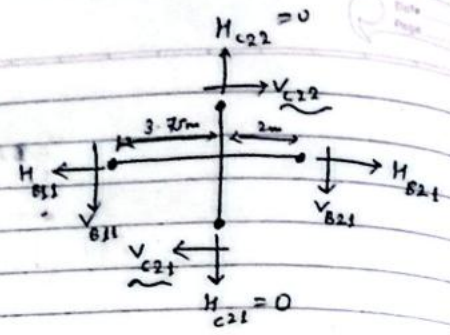
$[H_{B2} = 69.565 \text{ kN (C)}]$

(iii) $\sum F_y = 0$

$[V_{B2} = -250.405 \text{ kN}]$

Pinned

Mid span



$\sum M = 0$ (at any pt. of contraflexure of beam)

eqⁿ in form of V_{C22} & $V_{C11} = 0$

$\sum F_x = 0$
eqⁿ in V_{C22} & $V_{C11} = 0$

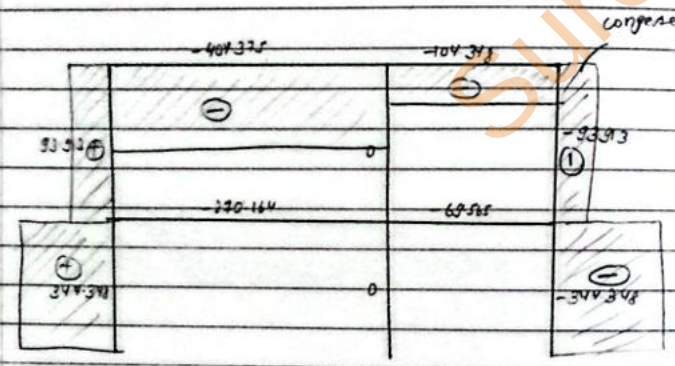
Solving

$V_{C11} = 300 \text{ kN}$

$V_{C22} = 500 \text{ kN}$

NOW

Axial force diagram:



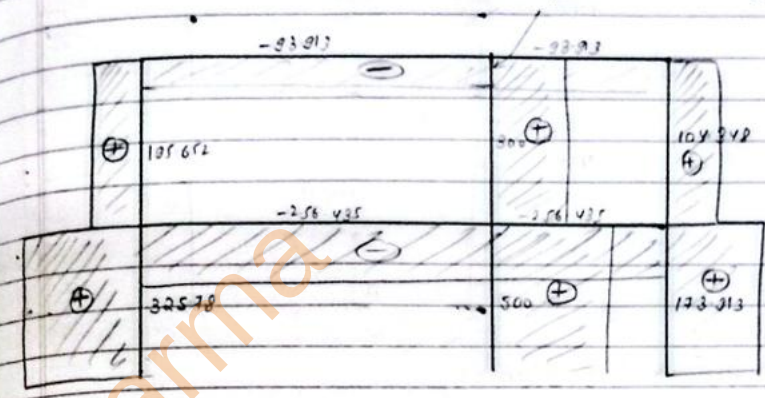
Compressed & Tension effects

AED/SFD Frame को निम्नलिखित वैसे

GMD → can do any

वर्ग → +ve
वर्ग → -ve

Shear force diagram

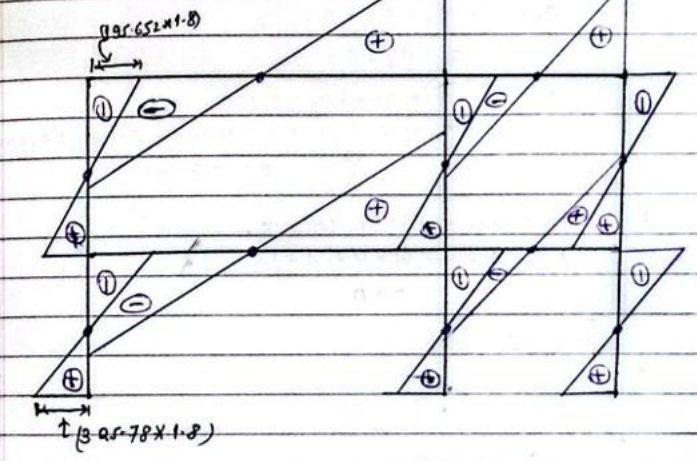
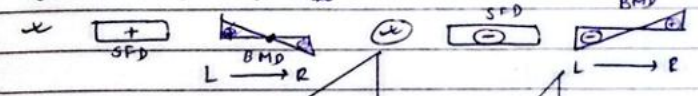


for more column 1st 2nd 3rd 4th

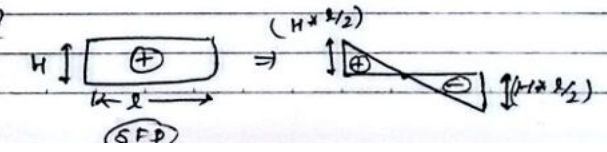
Bending Moment diagram

Based on SFD & assumptions (i) & (ii)

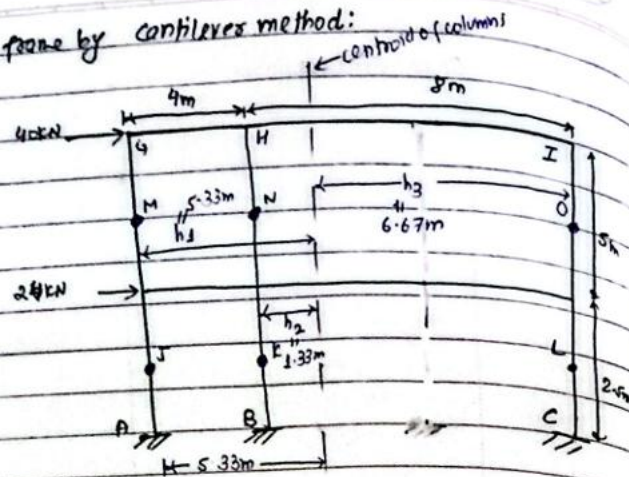
∝ SFD → BMD →



value??



(9) Analyze the frame by cantilever method:



Centroid of columns
 $\propto AF$ vary with distance of particular column from centroid of columns
 $AF \rightarrow$ vary linearly with the distance from centroid.

Sol:-

Centroid:

$$\bar{X} = \frac{A_1 \times X_1 + A_2 \times X_2 + A_3 \times X_3}{A_1 + A_2 + A_3}$$

$$A_1 + A_2 + A_3$$

$A \rightarrow$ Area of column

$X \rightarrow$ distance of column from defined point/origin

Assume, same area of column,

$$\bar{X} = \frac{A \times 0 + A \times 4 + A \times 12}{3 \times A}$$

$$[\bar{X} = 5.33m]$$

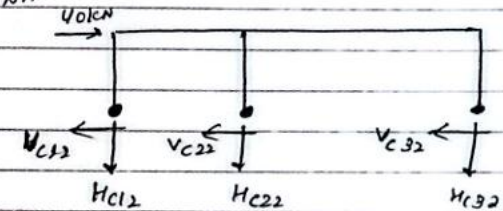
Form assumption

$$\frac{H_{C12}}{5.33} = \frac{H_{C22}}{1.33} = \frac{H_{C32}}{6.67} = a$$

$$H_{C22} = \frac{1.33}{5.33} \times H_{C12} \quad \text{--- (1)}$$

$$H_{C32} = \frac{6.67}{5.33} \times H_{C12} \quad \text{--- (2)}$$

Now same as portal frame,
 Consider upper section



$$\sum \Sigma M = 0$$

$$0, \quad 40 \times 2.5 + H_{C22} \times 4 + H_{C32} \times 12 = 0$$

$$= 1 \quad [H_{C12} = 7.141kN]$$

$$[H_{C22} = 1.791kN]$$

$$[H_{C32} = 8.93kN]$$

AF of all column in 2nd floor

Next

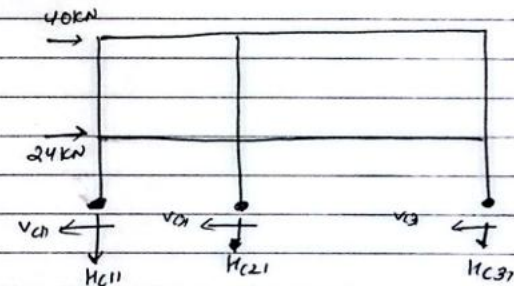
Similar above

$$\frac{H_{C11}}{5.33} = \frac{H_{C21}}{1.33} = \frac{H_{C31}}{6.67} = 0$$

$$H_{C11} \text{ \& } H_{C31} \rightarrow 0 \quad (H_{C11})$$

$$\sum \Sigma M = 0$$

$$[H_{C11} = 0]$$



Spectral Displacement Response (S_d)

- In standard eqn erg, S_d is used to represent true displacement
- NO, pseudo-displacement spectrum is defined independently.
- $S_d = \max |\dot{v}(t)|$; where $\dot{v}(t) =$ relative disp. response
- Spectral displacement is max. relative displacement of SDOF system subjected to given ground motion

*chr

- obtained directly from time history
- represents true physical displacement
- Most fundamental spectral qty
- used in displacement design & drift checks
- Spectral displacement derived from pseudo-velocity response & pseudo-acclⁿ response are just true spectral disp. response

$$S_d = \frac{S_{pv}}{\omega} = \frac{S_{pa}}{\omega^2}$$

H Spec vel. response (S_v)

- Maxⁿ rel. vel. of an SDOF system subjected to a given ground motion.
- $$S_v = \max |\dot{v}(t)|$$
- $\dot{v}(t) =$ rel. vel. response

*chr

- ob. "
- rep. " actual phy. velocity
- imp. for damping & energy studies
- depends on damping & frequency
- Spectral velocity calculated from spectral displacement or pseudo-acclⁿ spectral response are pseudo-velocity spectral response.

$$\text{i.e. } S_{pv} = \omega S_d \quad \& \quad S_{pv} = \frac{S_{pa}}{\omega}$$

→ True spectral velocity is rarely provided. Spv is very good approx.

Spectral Acceleration/Force response:

- Relative accel = → not used in design
- Absolute accel (used for inertial forces)
↳ used in design.

$$\begin{cases} S_{pa} = \omega S_{pv} \\ S_{pa} = \omega^2 S_d \end{cases}$$

- For linear system with low damping,
 $S_{pa} \approx S_a$ (true abs response)
- Most design codes use, S_{pa}

⇒ Conclusion

Spectral disp. response → S_d → true → relative disp.

" vel. " → $S_{pv} = \omega S_d$ → Pseudo (usually) → Approx relative vel.

" accel = " → $S_{pa} = \omega^2 S_d$ → Pseudo (absolute) → Used for inertial forces

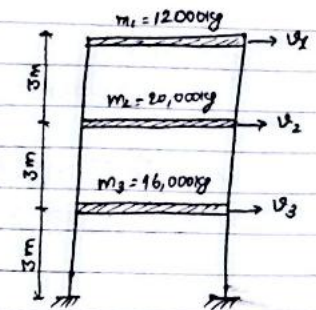
6B

Ch 3
Eg 1

A 3 storey building shown in figure below has the following vibration properties.

$$[\Phi] = \begin{bmatrix} 1 & 1 & 1 \\ 0.845 & -0.281 & -1.265 \\ 0.471 & -0.962 & 1.242 \end{bmatrix}$$

$$\{\omega\} = \begin{Bmatrix} 3.60 \\ 10.33 \\ 13.74 \end{Bmatrix} \text{ rad/sec}$$



Determine the displacement, overturning moments, shear force at each story and the base shear at time t_2 during an eqk. when the eqk. response integral for 3 modes are

$$\{\ddot{v}(t)\} = \begin{Bmatrix} 0.630 \\ -0.229 \\ 0.343 \end{Bmatrix} \text{ m/s}^2$$

→ Soln:-

1. Mass matrix,

$$[m] = 4 \times 10^3 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \text{ kg}$$

Mode shapes:

$$\{\phi_1\} = \begin{Bmatrix} 1 \\ 0.845 \\ 0.471 \end{Bmatrix}, \{\phi_2\} = \begin{Bmatrix} 1 \\ -0.281 \\ -0.962 \end{Bmatrix}, \{\phi_3\} = \begin{Bmatrix} 1 \\ -1.265 \\ 1.242 \end{Bmatrix}$$

Modal participation factors:

$$\lambda_n = \frac{L_n}{M_n} = \frac{\{\phi_n\}^T [m] \{1\}}{\{\phi_n\}^T [m] \{\phi_n\}}$$

$$L_1 = \{\phi_1\}^T [m] \{1\}$$

$$= \begin{bmatrix} 1 & 0.845 & 0.471 \end{bmatrix} 4 \times 10^3 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$= 36436 \text{ kg}$$

$$M_1 = \{\phi_1\}^T [m] \{\phi_1\}$$

$$= \begin{bmatrix} 1 & 0.845 & 0.471 \end{bmatrix} 4 \times 10^3 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{Bmatrix} 1 \\ 0.845 \\ 0.471 \end{Bmatrix}$$

$$= 29829.956 \text{ kg}$$

Similarly,

$$L_2 = \{\phi_2\}^T [m] \{1\} = -9012 \text{ kg}$$

$$M_2 = \{\phi_2\}^T [m] \{\phi_2\} = 28386.324 \text{ kg}$$

$$\begin{aligned} L_3 &= \{\phi_3\}^T [m] \{1\} = 6572 \text{ kg} \\ M_3 &= \{\phi_3\}^T [m] \{\phi_3\} = 68685.524 \text{ kg} \end{aligned}$$

Thus,

$$\lambda_1 = \frac{L_1}{M_1} = 1.2215$$

$$\lambda_2 = \frac{L_2}{M_2} = -0.3175$$

$$\lambda_3 = \frac{L_3}{M_3} = 0.0957$$

$$F = 1000 \text{ N ft/s}^2$$

* Storey displacements:

$$\{v_n(t_1)\} = \{\phi_n\} \frac{L_n}{M_n} \cdot \frac{v_n(t_1)}{\omega_n} \rightarrow \text{Modal displacement}$$

$$\{v(t_1)\} = [\phi] \left\{ \frac{L_n}{M_n} \cdot \frac{v_n(t_1)}{\omega_n} \right\} \rightarrow \text{total displacement}$$

For mode 1:

$$\{v_1(t_1)\} = \begin{Bmatrix} v_{11}(t_1) \\ v_{21}(t_1) \\ v_{31}(t_1) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0.845 \\ 0.471 \end{Bmatrix} \times 1.2215 \times \frac{0.630}{3.60} = \begin{Bmatrix} 0.214 \\ 0.181 \\ 0.101 \end{Bmatrix} \text{ m}$$

For mode 2:

$$\{v_2(t_1)\} = \begin{Bmatrix} v_{12}(t_1) \\ v_{22}(t_1) \\ v_{32}(t_1) \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0.281 \\ -0.962 \end{Bmatrix} \times -0.3175 \times \frac{-0.229}{10.33} = \begin{Bmatrix} 0.007 \\ -0.002 \\ -0.007 \end{Bmatrix} \text{ m}$$

For mode 3:

$$\{v_3(t_1)\} = \begin{Bmatrix} v_{13}(t_1) \\ v_{23}(t_1) \\ v_{33}(t_1) \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1.265 \\ 1.242 \end{Bmatrix} \times 0.0957 \times \frac{0.343}{13.74} = \begin{Bmatrix} 0.002 \\ -0.003 \\ 0.003 \end{Bmatrix} \text{ m}$$

\(\therefore\) Total displacement at time (\(t_1\))

$$\{v(t_1)\} = \{v_1(t_1)\} + \{v_2(t_1)\} + \{v_3(t_1)\} = \begin{Bmatrix} 0.223 \\ 0.176 \\ 0.097 \end{Bmatrix} \text{ m}$$

'OR'

checking (alternatively)

$$\{v(t)\} = [\Phi] \left\{ \frac{L_n}{M_n} \cdot \frac{w_n(t)}{\omega_n} \right\}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0.845 & -0.211 & -1.265 \\ 0.471 & -0.962 & 1.242 \end{bmatrix} \left\{ \begin{array}{l} 1.2215 \times \frac{0.63}{3.60} \\ -0.3175 \times \frac{-0.229}{10.33} \\ 0.0957 \times \frac{0.343}{13.74} \end{array} \right\}$$

$$= \begin{Bmatrix} 0.223 \\ 0.176 \\ 0.097 \end{Bmatrix} m$$

(O.K.)

* Lateral force/Elastic forces at each storey

$$\{f_{sn}(t)\} = [m] [\Phi_n] \frac{L_n \cdot \omega_n v_n(t)}{M_n} \rightarrow \text{Modal force}$$

$$\{f_s(t)\} = [m] [\Phi] \left\{ \frac{L_n}{M_n} \cdot \omega_n v_n(t) \right\} \rightarrow \text{Total force}$$

Mode 1:

$$\{f_{s1}(t)\} = \begin{Bmatrix} f_{s1}(t) \\ f_{s2}(t) \\ f_{s3}(t) \end{Bmatrix} = 4 \times 10^3 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{Bmatrix} 1 \\ 0.845 \\ 0.471 \end{Bmatrix} \times 1.2215 \times \frac{0.63 \times 3.60}{10.33}$$

$$= \begin{Bmatrix} 33244.344 \\ 46819.178 \\ 20877.44103 \end{Bmatrix} N$$

Similarly

Mode 2:

$$\{f_{s2}(t)\} = \begin{Bmatrix} f_{s1}(t) \\ f_{s2}(t) \\ f_{s3}(t) \end{Bmatrix} = \begin{Bmatrix} 9012.8217 \\ -4221.00483 \\ -11860.44587 \end{Bmatrix} N$$

Mode 3:

$$\{f_{s3}(t)\} = \begin{Bmatrix} f_{s1}(t) \\ f_{s2}(t) \\ f_{s3}(t) \end{Bmatrix} = \begin{Bmatrix} 5412.20488 \\ -11410.72691 \\ 8962.60732 \end{Bmatrix} N$$

Hence, total elastic forces at each storeys

$$\{f_s(t)\} = \{f_{s1}(t)\} + \{f_{s2}(t)\} + \{f_{s3}(t)\}$$

$$= \begin{Bmatrix} 47669.368 \\ 31187.386 \\ 18279.609 \end{Bmatrix} N$$

'OR'

checking (alternatively)

$$\{f_s(t)\} = [m] [\Phi] \left\{ \frac{L_n}{M_n} \cdot \omega_n v_n(t) \right\}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 0.845 & -0.211 & -1.265 \\ 0.471 & -0.962 & 1.242 \end{bmatrix} \left\{ \begin{array}{l} 1.2215 \times 0.63 \times 3.60 \\ -0.3175 \times -0.229 \times 10.33 \\ 0.0957 \times 0.343 \times 13.74 \end{array} \right\}$$

$$= \begin{Bmatrix} 47669.368 \\ 31187.386 \\ 18279.609 \end{Bmatrix} N$$

(O.K.)

* Shear force at each storey
(at time t_1)

$$V_1 = 47669.368 \text{ N} \approx 47.670 \text{ kN (Upper storey)}$$

$$V_2 = 47669.368 + 31187.386 = 78856.754 \text{ N} \\ \approx 78.857 \text{ kN (Mid-storey)}$$

$$V_3 = 78856.754 + 18279.609 = 97136.363 \text{ N} \\ \approx 97.136 \text{ kN (Base storey)}$$

* Base shear at time t_1 :

$$V_b = V_3 \text{ (shear force at base storey)} \\ = 97.136 \text{ kN}$$

* Overturning moment at each storey (at time t_1)

$$M_{01}(t_1) = 9 \times 47.670 = 429.03 \text{ kN-m (Upper storey)} \\ \text{check}$$

$$M_{02}(t_1) = 6 \times 47.670 + 9 \times 31.1874 = 379.5822 \text{ kN-m} \\ \text{(Mid-storey)}$$

$$M_{03}(t_1) = 9 \times 47.670 + 6 \times 31.1874 + 9 \times 18.280 \\ = 670.8844 \text{ kN-m} \\ \text{(Base storey)}$$

Alternatively,

Overturning moment at base at time t_1

$$M_0(t_1) = [h_i]^T [F(t_1)] \\ = [9 \quad 6 \quad 3] \begin{Bmatrix} 47669.368 \\ 31187.386 \\ 18279.609 \end{Bmatrix}$$

$$= 670.887455 \text{ kN-m} \\ \approx 670.8875 \text{ kN-m}$$

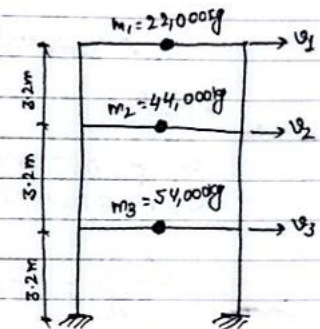
Q2 For a 3-storey building shown, determine for each mode of vibration, the max displacements, lateral force, shear force, and overturning moments at each storey level. Also, determine total maximums due to all modes using SRS method.

$$[\Phi] = \begin{bmatrix} 1 & 1 & 1 \\ 0.51 & -1.50 & -6.24 \\ 0.20 & -0.87 & 14.12 \end{bmatrix}$$

$$\{\omega\} = \begin{Bmatrix} 4.89 \\ 8.50 \\ 14.20 \end{Bmatrix} \text{ rad/sec}$$

pseudo-spectral velocity responder

$$\{S_v\} = \begin{Bmatrix} 0.65 \\ 0.24 \\ 0.37 \end{Bmatrix} \text{ m/sec}$$



→ Soln:-

$$\text{Mass matrix, } [m] = 2 \times 10^3 \begin{bmatrix} 11 & 0 & 0 \\ 0 & 22 & 0 \\ 0 & 0 & 27 \end{bmatrix} \text{ kg}$$

Mode shapes

$$\{\Phi_1\} = \begin{Bmatrix} 1 \\ 0.51 \\ 0.20 \end{Bmatrix}, \{\Phi_2\} = \begin{Bmatrix} 1 \\ -1.50 \\ -0.87 \end{Bmatrix}, \{\Phi_3\} = \begin{Bmatrix} 1 \\ -6.24 \\ 14.12 \end{Bmatrix}$$

Modal participation factors

$$\lambda_n = \frac{L_n}{m_n} = \frac{\{\Phi_n\}^T [m] \{\Phi_n\}}{\{\Phi_n\}^T [m] \{\Phi_n\}}$$

$$L_1 = \{\phi_1\}^T [m] \{\phi_1\} \\ = [1 \ 0.51 \ 0.20] 2 \times 10^3 \begin{bmatrix} 11 & 0 & 0 \\ 0 & 22 & 0 \\ 0 & 0 & 27 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$= 5.524 \times 10^4 \text{ kg}$$

$$M_1 = \{\phi_1\}^T [m] \{\phi_1\} \\ = 2 [1 \ 0.51 \ 0.20] 2 \times 10^3 \begin{bmatrix} 11 & 0 & 0 \\ 0 & 22 & 0 \\ 0 & 0 & 27 \end{bmatrix} \begin{Bmatrix} 1 \\ 0.51 \\ 0.20 \end{Bmatrix}$$

$$= 3.56044 \times 10^4 \text{ kg}$$

Similarly,

$$\left. \begin{aligned} L_2 &= -9.098 \times 10^4 \text{ kg} \\ M_2 &= 16.18726 \times 10^4 \text{ kg} \end{aligned} \right\} \begin{aligned} L_3 &= 50.992 \times 10^4 \text{ kg} \\ M_3 &= 1250.1472 \times 10^4 \text{ kg} \end{aligned}$$

$$\therefore \lambda_1 = \frac{L_1}{M_1} = 1.552$$

$$\lambda_2 = \frac{L_2}{M_2} = -0.562$$

$$\lambda_3 = \frac{L_3}{M_3} = 0.041$$

* Max = Model displacement

$$\{u_{n, \max}\} = \{\phi_n\} = \frac{L_n}{M_n} \cdot \frac{S_{u,n}}{\omega_n}$$

Model 1:

$$\{u_{1, \max}\} = \begin{Bmatrix} u_{11, \max} \\ u_{21, \max} \\ u_{31, \max} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0.51 \\ 0.20 \end{Bmatrix} \times \frac{1.552 \times 0.65}{4.29} = \begin{Bmatrix} 0.206 \\ 0.105 \\ 0.041 \end{Bmatrix} \text{ m}$$

Model 2:

$$\{u_{2, \max}\} = \begin{Bmatrix} u_{12, \max} \\ u_{22, \max} \\ u_{32, \max} \end{Bmatrix} = \begin{Bmatrix} -0.016 \\ 0.024 \\ 0.014 \end{Bmatrix} \text{ m}$$

Model 3:

$$\{u_{3, \max}\} = \begin{Bmatrix} u_{13, \max} \\ u_{23, \max} \\ u_{33, \max} \end{Bmatrix} = \begin{Bmatrix} 0.001 \\ -0.007 \\ 0.015 \end{Bmatrix} \text{ m}$$

\therefore Total max = displacement using SRS is:-

$$\{u_{\max}\} = \begin{Bmatrix} [u_{11, \max}^2 + u_{12, \max}^2 + u_{13, \max}^2]^{1/2} \\ [u_{21, \max}^2 + u_{22, \max}^2 + u_{23, \max}^2]^{1/2} \\ [u_{31, \max}^2 + u_{32, \max}^2 + u_{33, \max}^2]^{1/2} \end{Bmatrix}$$

$$= \begin{Bmatrix} [0.206^2 + 0.016^2 + 0.001^2]^{1/2} \\ [0.105^2 + 0.024^2 + 0.007^2]^{1/2} \\ [0.041^2 + 0.014^2 + 0.015^2]^{1/2} \end{Bmatrix}$$

$$= \begin{Bmatrix} 0.207 \\ 0.108 \\ 0.046 \end{Bmatrix} \text{ m}$$

* Max = model lateral force (Elastic force)

$$\{f_{sn, \max}\} = [m] \{\phi_n\} \cdot \frac{L_n}{M_n} \cdot \omega_n \cdot S_{u,n}$$

Model 1:

$$\{f_{s1, \max}\} = \begin{Bmatrix} f_{s11, \max} \\ f_{s21, \max} \\ f_{s31, \max} \end{Bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 22 & 0 \\ 0 & 0 & 27 \end{bmatrix} 2 \times 10^3 \times 1.552 \times 4.29 \times \begin{Bmatrix} 1 \\ 0.51 \\ 0.20 \end{Bmatrix} \\ = \begin{Bmatrix} 108.527 \\ 110.697 \\ 53.278 \end{Bmatrix} \text{ kN}$$

Mode 2:

$$\{f_{s1,max}\} = \begin{Bmatrix} -25.223 \\ 75.668 \\ 53.862 \end{Bmatrix} \text{ kN}$$

Mode 3:

$$\{f_{s3,max}\} = \begin{Bmatrix} 4.739 \\ -59.144 \\ 164.249 \end{Bmatrix} \text{ kN}$$

∴ Total maximum lateral force using SRSS is:

$$\{f_{s,max}\} = \begin{Bmatrix} [f_{s1,max}^2 + f_{s2,max}^2 + f_{s3,max}^2]^{1/2} \\ [f_{s2,max}^2 + f_{s22,max}^2 + f_{s23,max}^2]^{1/2} \\ [f_{s3,max}^2 + f_{s32,max}^2 + f_{s33,max}^2]^{1/2} \end{Bmatrix} = \begin{Bmatrix} 111.579.873 \\ 146552.04 \\ 180878.902 \end{Bmatrix} \text{ N}$$

* Max² Modal shear force:

Mode 1:

$$\{V_{1,max}\} = \begin{Bmatrix} V_{11,max} \\ V_{21,max} \\ V_{31,max} \end{Bmatrix} = \begin{Bmatrix} 108.527 \\ 110.897 + 108.527 \\ 53.278 + 110.897 + 108.527 \end{Bmatrix} = \begin{Bmatrix} 108.527 \\ 219.244 \\ 2725.02 \end{Bmatrix} \text{ kN}$$

Sim

Mode 2:

$$\{V_{2,max}\} = \begin{Bmatrix} -25.223 \\ 50.445 \\ 104.367 \end{Bmatrix} \text{ kN}$$

Mode 3:

$$\{V_{3,max}\} = \begin{Bmatrix} 4.739 \\ -54.405 \\ 105.844 \end{Bmatrix} \text{ kN}$$

∴ Total max² shear using SRSS is:

$$\{V_{max}\} = \begin{Bmatrix} 111.520 \\ 231.438 \\ 311.774 \end{Bmatrix} \text{ kN}$$

* Max² Modal overturning moment

Mode 1:

$$\{M_{1,max}\} = \begin{Bmatrix} M_{11,max} \\ M_{21,max} \\ M_{31,max} \end{Bmatrix} = \begin{Bmatrix} 108.527 \times 3.2 \\ 108.527 \times 6.4 + 110.897 \times 3.2 \\ 108.527 \times 9.6 + 110.897 \times 6.4 + 53.278 \times 3.2 \end{Bmatrix} = \begin{Bmatrix} 347.286 \\ 1048.803 \\ 1920.80 \end{Bmatrix} \text{ kN.m}$$

Sim

Mode 2:

$$\{M_{2,max}\} = \begin{Bmatrix} -80.714 \\ 80.710 \\ 414.453 \end{Bmatrix} \text{ kN.m}$$

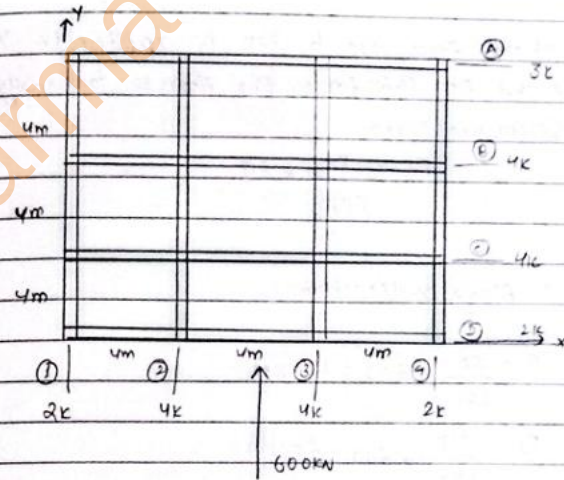
$$\{M_{3,max}\} = \begin{Bmatrix} 15.165 \\ -158.931 \\ 192.570 \end{Bmatrix} \text{ kN.m}$$

∴ Total Max² overturning moment using SRSS is:

$$\{M_{1,max}\} = \begin{Bmatrix} 356.865 \\ 1063.843 \\ 1974.436 \end{Bmatrix} \text{ kN.m}$$

(9)

The figure given below shows the plan of an one-storey building which could be considered as composed of 2D frames along the orthogonal directions. The roof diaphragm is rigid in its own plane, and the mass of the roof uniformly distributed. The building is subjected to a lateral load of 600kN, due to eqk in Y-direction, and passing through the centre of mass of the building. Calculate the lateral force in 2D frames along X and Y direction.



→ Solⁿ:-

* Location of centre of mass (CM)

Since the mass is uniformly distributed and plan is rectangular; the centre of mass coincides with geometric centroid. Hence

$$(X_m, Y_m) = (6m, 6m)$$

* Location of centre of Rigidity (CR)

As diaphragm is rigid; hence

$$X_p = \frac{\sum k_i x_i}{\sum k_i} = \frac{2k \times 0 + 4k \times 4 + 4k \times 8 + 2k \times 12}{2k + 4k + 4k + 2k} = 6m$$

$$Y_p = \frac{\sum x_i y_i}{\sum x_i} = \frac{2k \times 0 + 4k \times 4 + 4k \times 8 + 3k \times 12}{(2k + 4k + 4k + 3k)} = 6.46m$$

$$\therefore (x_p, y_p) = (6m, 6.46m)$$

Now,

$$\text{eccentricity along X-direction} = x_m - x_p = 0m$$

$$\text{" " Y-direction} = y_m - y_p = 6 - 6.46 = -0.46m$$

A lateral force due to eqk is applied in Y-direction only and for this force, the floor is torsionally uncoupled system, we have

$$F_i = \frac{x_i}{\sum x_i} \times EQ$$

$\therefore$ Along Y-direction:

$$F_1 = \frac{2k}{12k} \times 600 = 100kN$$

$$F_2 = \frac{4k}{12k} \times 600 = 200kN$$

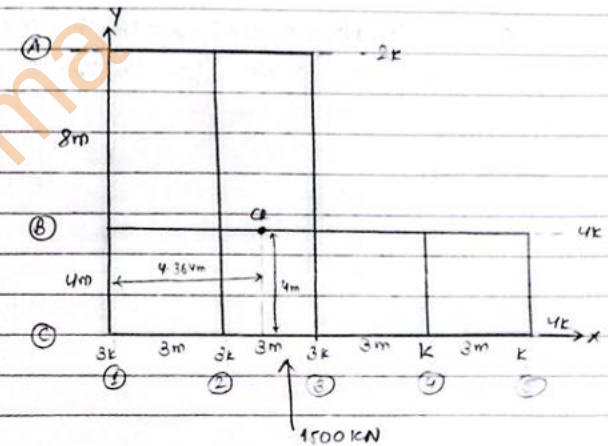
$$F_3 = \frac{4k}{12k} \times 600 = 200kN$$

$$F_4 = \frac{2k}{12k} \times 600 = 100kN$$

As there is no lateral force applied in X-direction and for lateral force in Y-dirⁿ, the floor system is torsionally uncoupled; hence

$$F_A = F_B = F_C = F_D = 0kN.$$

(Q.2) The figure shows the plan of one storey building which could be considered as composed of 2D frames along orthogonal directions. The roof diaphragm is rigid in its own plane, and the mass of the roof is uniformly distributed. The building is subjected to a lateral load of 1500kN due to eqk. in X-direction and passing through centre of mass of the building. Calculate the lateral forces in the 2D frames along X as well as Y-direction.



$\rightarrow$ Solⁿ.

* CM

$$x_m = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6 \times 12) \times 3 + (6 \times 4 \times m) \times 9}{(6 \times 12 \times m + 6 \times 4 \times m)} = 4.5m$$

$m = \text{mass per unit area}$

$$y_m = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6 \times 12 \times m) \times 6 + (6 \times 4 \times m) \times 2}{(6 \times 12 \times m + 6 \times 4 \times m)} = 5m$$

$$\therefore (x_m, y_m) = (4.5m, 5.0m)$$

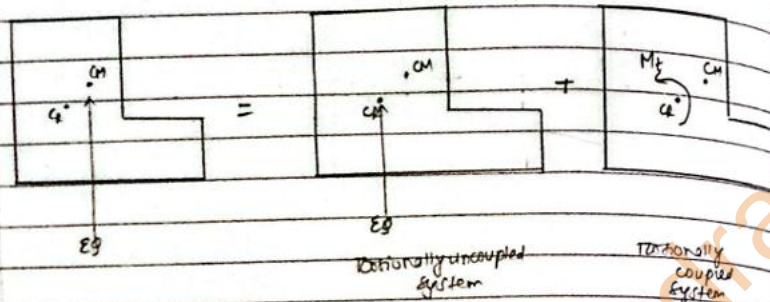
* CR

$$X_R = \frac{\sum k_i x_i}{\sum k_i} = \frac{3k \times 0 + 3k \times 3 + 3k \times 6 + k \times 9 + k \times 12}{(3k + 3k + 3k + k + k)} = 4.364m$$

$$Y_R = \frac{\sum k_i y_i}{\sum k_i} = \frac{4k \times 0 + 4k \times 4 + 2k \times 12}{(4k + 4k + 2k)} = 4m$$

$$\therefore (X_R, Y_R) = (4.364m, 4m)$$

Eccentricity along X-axis, $e_x = X_m - X_R = 4.5 - 4.364 = 0.136m$
 " " Y-axis, $e_y = Y_m - Y_R = 5 - 4 = 1m$



* lateral forces due to torsionally uncoupled system

$$F_i = \frac{k_i}{\sum k_i} \times EQ$$

$$F_1 = \frac{3k}{11k} \times 1500 = 409.091kN (\downarrow)$$

$$F_2 = F_3 = 409.091kN (\downarrow)$$

$$F_4 = F_5 = \frac{k}{11k} \times 1500 = 136.364kN (\downarrow)$$

* lateral forces due to torsionally coupled system.

$$\text{Torsional moment, } M_t = EQ \times e_y$$

$$= 1500 \times 0.136$$

$$= 204kNm$$

$$F_i = \frac{k_i e_i}{\sum k_i e_i^2} \times M_t$$

Frame	k_i	$e_i (m)$	$k_i e_i$	$k_i e_i^2$	$\frac{k_i e_i}{\sum k_i e_i^2}$	$F_i (kN)$
①	3k	4.364	13.092k	57.133k		7.797 (↑)
②	3k	1.364	4.092k	5.581k		2.437 (↑)
③	3k	1.636	4.908k	8.029k		2.923 (↓)
④	k	4.636	4.636k	21.492k		2.761 (↓)
⑤	k	7.636	7.636k	58.308k		4.548 (↓)
⑥	2k	8	16k	128k		9.529 (→)
⑦	4k	0	0	0		0
⑧	4k	4	16k	64k		9.529 (←)

$$[\sum k_i e_i^2 = 842.843k]$$

↑
to produce
existing
2 moment

∴ Total forces in frames due to EQ

$$F_1 = 409.091 - 7.797 = 401.294kN (\downarrow)$$

$$F_2 = 409.091 - 2.437 = 406.654kN (\downarrow)$$

$$F_3 = 409.091 + 2.923 = 412.014kN (\downarrow)$$

$$F_4 = 136.364 + 2.761 = 139.125kN (\downarrow)$$

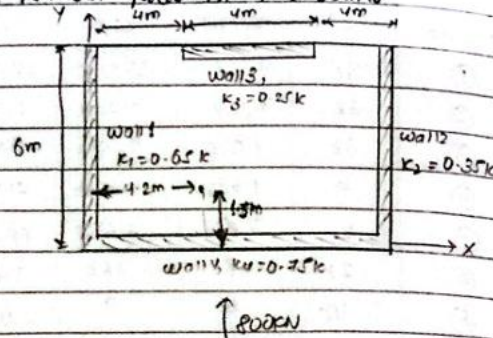
$$F_5 = 136.364 + 4.548 = 140.912kN (\downarrow)$$

$$F_6 = 9.529kN (\rightarrow)$$

$$F_7 = 0$$

$$F_8 = 9.529kN (\leftarrow)$$

Q.3 The fig. given shows the plan of a one-storey building supported on four shear walls with the lateral stiffness as indicated, and oriented as shown. The roof diaphragm is rigid in its own plane, and the mass of the roof is uniformly distributed. The building is subjected to a lateral load of 800 kN, due to eq. in Y-direction and passing through the centre of mass of building. Calculate the lateral forces in the walls.



→ Soln:-

* CM

$$(x_m, y_m) = (6m, 3m)$$

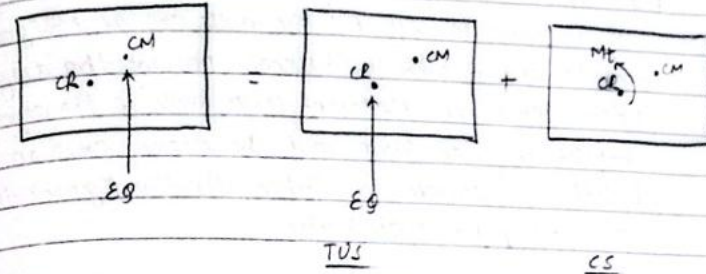
* CR

$$x_p = \frac{\sum k_i x_i}{\sum k_i} = \frac{0.65k \times 0 + 0.35k \times 12}{(0.65k + 0.35k)} = 4.2m$$

$$y_p = \frac{\sum k_i y_i}{\sum k_i} = \frac{0.75k \times 0 + 0.25k \times 6}{(0.75k + 0.25k)} = 1.5m$$

$$\therefore (x_p, y_p) = (4.2m, 1.5m)$$

* Eccentricity along x-dirⁿ (e_m) $x_m - x_p = 6 - 4.2 = 1.8m$



* Lateral force due to translationally uncoupled system

$$F_i = \frac{k_i}{\sum k_i} \times E \cdot g$$

$$F_1 = \frac{0.65k}{k} \times 800 = 520kN (\downarrow)$$

$$F_2 = \frac{0.35k}{k} \times 800 = 280kN (\downarrow)$$

* Lateral force due to torsionally coupled system

$$M_t = 800 \times 1.8 = 1440 kN \cdot m$$

$$F_i = \frac{k_i \cdot x_i}{\sum k_i \cdot x_i^2} \times M_t$$

Wall	k_i	$x_i(m)$	$k_i x_i$	$k_i x_i^2$	$F_i (kN)$
1	0.65k	4.2	2.73k	11.466k	99.50 (↑)
2	0.35k	7.8	2.73k	21.294k	99.50 (↓)
3	0.75k	4.5	1.125k	5.0625k	41 (→)
4	0.25k	1.5	1.125k	1.6875k	41 (←)

$$\sum k_i x_i^2 = 39.510k$$

* total forces in wall due to $E \cdot g$

$$F_1 = 520 - 99.50 = 420.50kN (\downarrow)$$

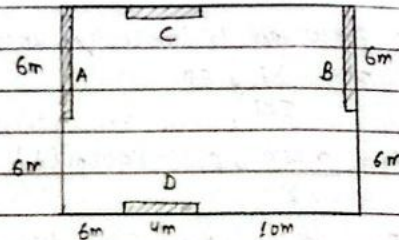
$$F_2 = 280 + 99.50 = 379.50kN (\downarrow)$$

$$F_3 = 41kN (\rightarrow)$$

$$F_4 = 41kN (\leftarrow)$$

me (9-4)

Plan of a building having four shear walls A, B, C and D as shown in fig. All the walls are of M25 grade concrete and 250mm in thickness. Describe the design lateral force on different shear walls, if the storey height is given 4m and the seismic force on the building is 200kN in either direction. Ignore the out of plane contribution.



→ Solⁿ:

Grade of concrete = M25

$$F_c = 5000 \sqrt{f_{ck}} = 25000 \text{ N/mm}^2$$

Thk of wall = 250mm

Storey ht (L) = 4m = 4000mm

Lateral stiffness of walls, $k = \frac{12EI}{L^3}$

± (k)

For walls A & B,

$$k_A = k_B = \frac{12E \times (250) \times (6000)^3}{12 \times (4000)^3} = \frac{3375E}{4} = 3.375E$$

±

For walls C & D,

$$k_C = k_D = \frac{12E \times (250) \times (4000)^3}{12 \times (4000)^3} = 250E = \frac{1000E}{4} \text{ (avg)}$$

CM

$$(X_m, Y_m) = (10m, 6m)$$

CR

Due to symmetry

$$(X_p, Y_p) = (10m, 6m)$$

iii) Earthquake force in x-direction

$$\text{Eccentricity } (e_y) = 6 - 6 = 0$$

x Lateral force in walls due to translation

$$F_i = \frac{k_i}{\sum k_i} \times E_0$$

$$F_C = F_D = \frac{k}{2k} \times 200 = 100 \text{ kN } (\leftarrow)$$

* Design eccentricity (Cl. 7.8.2 - IS 1893 (2016))

$$e_{di} = \begin{cases} 1.5 e_{si} + 0.05 b_i \\ e_{si} - 0.05 b_i \end{cases} \text{ whichever gives the more severe effect}$$

$$e_{d1} = \begin{cases} 1.5 \times 0 + 0.05 \times 12 = 0.6 \text{ m} \\ 0 - 0.05 \times 12 = -0.6 \text{ m} \end{cases}$$

$$\text{Torsional moment; } M_t = E_0 \times e_{di} = 200 \times (\pm 0.6) = \pm 120 \text{ kNm}$$

* Lateral force in wall due to torsional moment

$$F_i = \frac{k_i \alpha_i}{\sum k_i \alpha_i^2} \times M_t$$

Wall	k_i	$\alpha_i (m)$	$k_i \alpha_i$	$k_i \alpha_i^2$	$F_i (kN)$
A	3375k	10	3375k	3375k	5.422
B	3375k	10	3375k	3375k	5.422
C	k	6	6k	36k	0.964
D	k	6	6k	36k	0.964

$$\sum k_i \alpha_i^2 = 747k$$

∴ Total lateral force in wall due to seismic load in X-direction:

$$F_A = 5.422 kN$$

$$F_B = 5.422 kN$$

$$F_C = 100 \pm 0.964 = 100.964 kN$$

$$F_D = 100 \pm 0.964 = 100.964 kN$$

400 Earthquake force in Y-direction

* Lateral force due to translation

$$F_A = F_B = \frac{3375k}{6.75k} \times 200 = 100 kN (\downarrow)$$

* Lateral force due to torsional moment

$$\text{Eccentricity } e_x = 0$$

* Design eccentricity (IS 1893-2010)

$$e_{di} = \begin{cases} 1.5e_{hi} + 0.05b \\ e_{hi} - 0.05b \end{cases} \text{ whichever gives } \begin{cases} \text{adverse effect} \\ \text{favorable effect} \end{cases}$$

$$= \begin{cases} 1.5 \times 0 + 0.05 \times 20 = +1m \\ 0 - 0.05 \times 20 = -1m \end{cases} = \pm 1m$$

$$\text{Torsional moment, } M_t = 200k (\pm 1) = \pm 200 kNm$$

* Lateral force in wall due to torsional moment

$$F_i = \frac{k_i \alpha_i}{\sum k_i \alpha_i^2} \times M_t$$

Wall	k_i	$\alpha_i (m)$	$k_i \alpha_i$	$k_i \alpha_i^2$	$F_i (kN)$
A	3375k	10	3375k	3375k	9.036
B	3375k	10	3375k	3375k	9.036
C	k	6	6k	36k	1.606
D	k	6	6k	36k	1.606

$$[\sum k_i \alpha_i^2 = 747k]$$

∴ Total force in wall due to seismic force in Y-dir:

$$F_A = 100 \pm 9.036 = 109.036 kN (\text{max})$$

$$F_B = 100 \pm 9.036 = 109.036 kN (\text{max})$$

$$F_C = 1.606 kN$$

$$F_D = 1.606 kN$$

Hence, design lateral force on wall due to seismic loading either in X- or Y-direction are

$$F_A = 109.036 kN (\text{max})$$

$$F_B = 109.036 kN (\text{max})$$

$$F_C = 100.964 kN (\text{max})$$

$$F_D = 100.964 kN (\text{max})$$

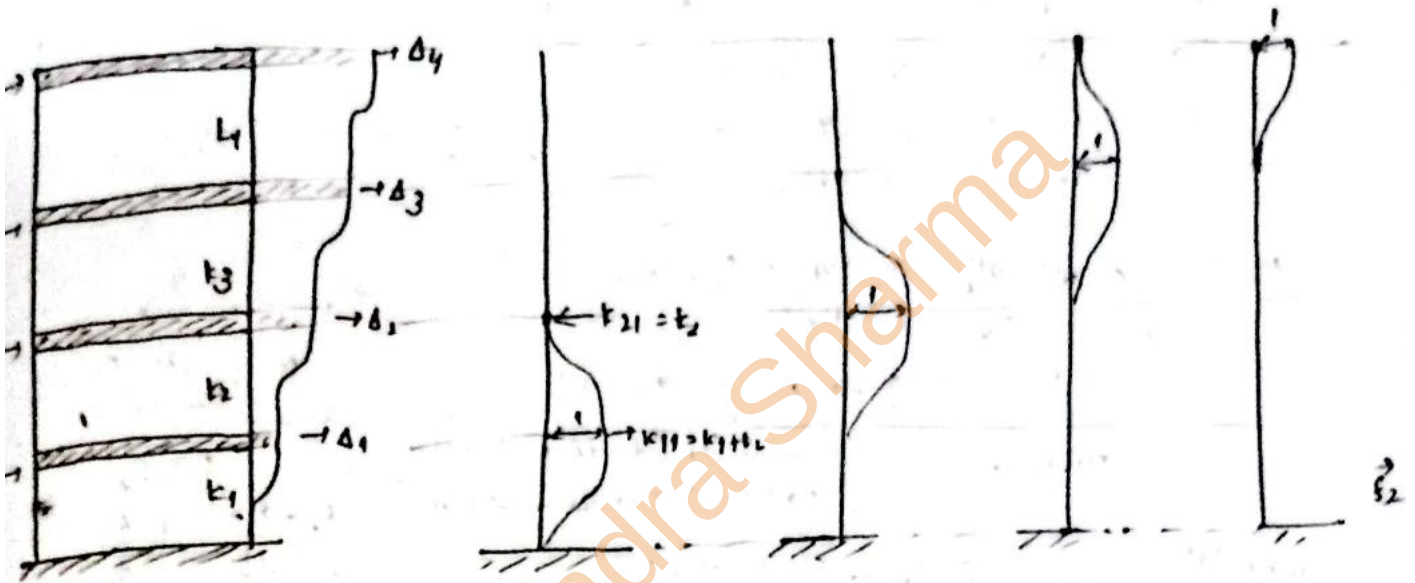
lateral stiffness matrix multistorey 2D frame

with infinitely rigid beams ($EI \rightarrow \infty$ for beams)

→ the beams do not bend → mean same rotation for both ends of beam.

→ frame behaves like a shear building with one lateral DOF

per storey → each storey has single lateral DOF



Stiffness matrix

$$[k] = \begin{bmatrix} k_{11} & k_{12} & 0 & 0 \\ k_{21} & k_{22} & k_{23} & 0 \\ 0 & k_{31} & k_{33} & k_{34} \\ 0 & 0 & k_{43} & k_{44} \end{bmatrix}$$

$$= \begin{bmatrix} [k_1 + k_2] & -k_2 & 0 & 0 \\ -k_2 + k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix}$$

$$\{P\} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix}$$

$$\{\Delta\} = \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{Bmatrix}$$

→ Force-displacement relation

$$\{P\} = [k]\{\Delta\}$$

$$\Rightarrow \{\Delta\} = [k]^{-1}\{P\}$$

Two storey 3D frame symmetrical in both x- & y-dir (with rigid diaphragm)

- columns & frames are placed symmetrically.
- Stiffness distribution is symmetric
- Mass " "
- CM coincides with CR
- No torsion/twisting when lateral load is applied
- x-dir & y-dir motion are independent.
- Torsionally uncoupled system.
- Hence can be analyzed separately for either orthogonal (x- or y-) direction

Ex. (torsionally uncoupled system)

Given E, I_y, P_1, P_2

$[K] \rightarrow$ along y-dir.

To find: Lateral load distribution & lateral forces in frames.

(Total lateral stiffness in y-dir)

$$[K] = [K]_A + [K]_B + [K]_C$$

$$\{F\} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

$$\{\Delta\} = \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix}$$

From $\{F\} = [C] \{\Delta\}$

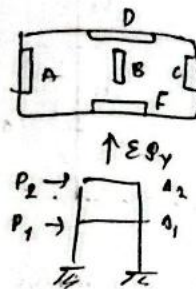
Building symmetric & torsionally uncoupled.

$$\{\Delta\} = \{\Delta\}_A = \{\Delta\}_B = \{\Delta\}_C$$

Then Lateral force in frame

$$\text{For diff. rel. } \begin{cases} \{F\}_A = [K]_A \{\Delta\}_A \\ \{F\}_B = [K]_B \{\Delta\}_B \end{cases}$$

$$\{F\}_A + \{F\}_B + \{F\}_C = \{P\} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \text{ kN}$$



Two storey 3D frame with rigid diaphragm

(General frame, not symmetrical)

- Stiffness dist. not symmetrical (Normal)
- Twisting/torsion of floor when lateral load applied.
- torsionally coupled system.
- Deformation & lateral load sharing by each frame can be analyzed separately for lateral load in either (x- or y-) dir.; and final results to be added for each frame or can be analyzed simultaneously.

* Generation of Global stiffness, force vector, displacement vector & individual frame stiffness matrix, disp vector & force vector

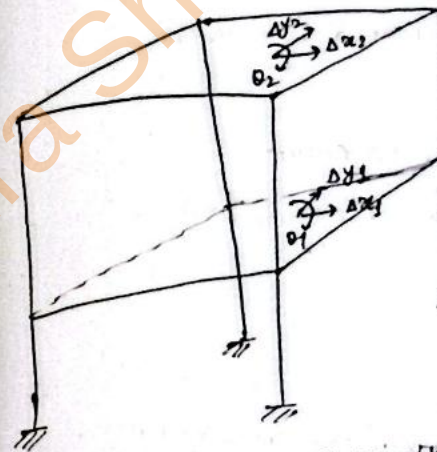


Fig: Typical 3D frame with global displacements

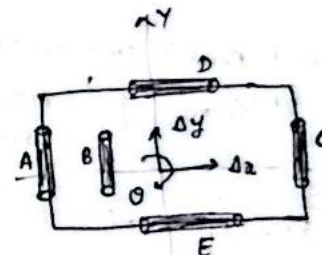


Fig: Typical plane with global displacement

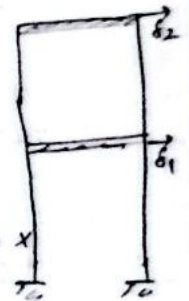


Fig: Frame B with local displacements

Global stiffness matrix,

$$[K]_{\text{global}} \rightarrow 6 \times 6$$

Local stiffness matrix for individual frame,

$$[K]_{\text{frame}} \rightarrow 2 \times 2$$

Global displacement vector for 3D frame

$$\{u\} = \begin{Bmatrix} \delta x_1 \\ \delta y_1 \\ \theta_1 \\ \delta x_2 \\ \delta y_2 \\ \theta_2 \end{Bmatrix}$$

$\delta x, \delta y \rightarrow$ lateral disp. along X- & Y- direction
 $\theta \rightarrow$ angle of twist

Local disp. vector for 2D frame

$$\{d\} = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$$

Global force vector for 3D frame

$$\{F\} = \begin{Bmatrix} F_{x1} \\ F_{y1} \\ M_1 \\ F_{x2} \\ F_{y2} \\ M_2 \end{Bmatrix}$$

$F_{x, y} \rightarrow$ lateral forces along X- & Y- at CE
 $M \rightarrow$ rotational moment

Individual frame force vector for 2D frame

$$\{F\}_{frame} = \begin{Bmatrix} F_{1, frame} \\ F_{2, frame} \end{Bmatrix} = [K]_{frame} \{d\}$$

We have

$$\{d\} = [B] \{u\}$$

2x1 2x6 6x1

where, $[B]$ displacement transformation matrix which transform 6 global disp. to 2 local disp. for individual frame
 $\rightarrow$ diff. value for diff. frame

Transformation matrix for individual frames

* For frame ABC (frame along Y-dir)

$$\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{[B]} \begin{Bmatrix} \delta x_1 \\ \delta y_1 \\ \theta_1 \\ \delta x_2 \\ \delta y_2 \\ \theta_2 \end{Bmatrix}$$

* For frame DEF (frame along X-dir)

$$\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}}_{[B]} \begin{Bmatrix} \delta x_1 \\ \delta y_1 \\ \theta_1 \\ \delta x_2 \\ \delta y_2 \\ \theta_2 \end{Bmatrix}$$

(distance of frame from CE)

* Global stiffness matrix

$\Rightarrow$ By individual frame

$$[K]_{global} = [B]^T [K]_{local} [B]$$

$$[K]_{global} = [B]_A^T [K]_n [B]_A$$

6x6 6x2 2x2 2x6

So on

$\Rightarrow$ For 3D frame

$$[K]_{global} = [K]_{A, global} + [K]_{B, global} + \dots + [K]_{E, global}$$

Global force-disp. relation:

$$\{F\} = [K]_{global} \{u\}$$

Q.1

classmate

Date
Page

Q.1 A five storey reinforced concrete public building with a total height of 15m and a storey height of 3m each has to be designed in special moment resisting frame. The proposed building is located in seismic zone IV and dead loads is 9 kN/m^2 on floors and 11 kN/m^2 at roof. The floors are subjected to live load of 4 kN/m^2 and roof to 2 kN/m^2 . If the floor and roof at each level have area of 360 m^2 , determine design seismic load on structure as per IS 1893 (part 1): 2016.

→ Solⁿ:-

Lumped dead load on each floor = 9 kN/m^2

" " " " roof = 11 kN/m^2

Live load on each floor = 4 kN/m^2

" " " " roof = 2 kN/m^2

Floor area in each storey = 360 m^2

Seismic zone : IV

Type of frame : SMRF

Soil condition : Medium

* Calculation of seismic wt. of each floor

* For first floor

(Cl. 7.3, 7.4 ; IS 1893:2016)

Dead load = $9 \times 360 = 3240 \text{ kN}$

Live load = $4 \times 360 \times (0.5) = 720 \text{ kN}$ [Cl. 7.3.1, Table 10].

$\therefore$ seismic weight (W_1) = $3240 + 720$
= 3960 kN

Similarly,

$W_2 = W_3 = W_4 = 3960 \text{ kN}$ (floors are similar)

* Seismic wt. of roof (5th floor)

$$\text{Dead load} = 11 \times 360 = 3960 \text{ kN}$$

$$\text{Live load} = 0 \text{ kN (Cl. 7.3.2)}$$

$$\therefore W_5 = 3960 \text{ kN}$$

* Total seismic weight of building

$$W = W_1 + W_2 + W_3 + W_4 + W_5$$

$$= 3960 \times 5$$

$$= 19,800 \text{ kN}$$

* Fundamental time period

(considering MRF without infill wall)

$$T_d = 0.075 h^{0.75} \quad (\text{Cl. 7.6.2})$$

$$= 0.075 \times (15)^{0.75}$$

$$= 0.572 \text{ sec.}$$

* Design horizontal seismic coefficient as per Cl. 6.4.2 of IS 1893: 2016 is given by

$$A_h = \frac{\left(\frac{Z}{2}\right) \left(\frac{S_a}{g}\right)}{\left(\frac{R}{S}\right)}$$

where,

$$Z = 0.36 \text{ (seismic zone II)} \quad [\text{Table 3}]$$

$$\frac{S_a}{g} = \frac{1.36}{T_d} \quad \text{for medium soil 4} \quad (\text{Cl. 6.4.2 (a), (b)})$$

$$0.55 < T_d < 4$$

$$= \frac{1.36}{0.572} = 2.378$$

$$0.572$$

R = Response reduction factor

$$= 5 \quad (\text{For S MRF}) \quad [\text{Table 9}]$$

I = Importance factor (Table 8)

$$= 1.5 \quad (\text{public building})$$

$$\therefore A_h = \frac{\left(\frac{0.36}{2}\right) \times 2.378}{\left(\frac{5}{1.5}\right)}$$

$$= 0.1285$$

* Design lateral force (base shear) [Cl. 7.2.1]

$$V_B = A_h \cdot W$$

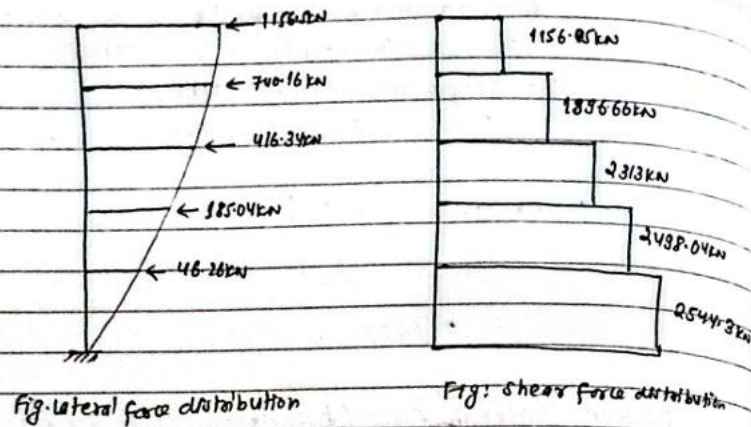
$$= 0.1285 \times 19800$$

$$[V_B = 2544.3 \text{ kN}]$$

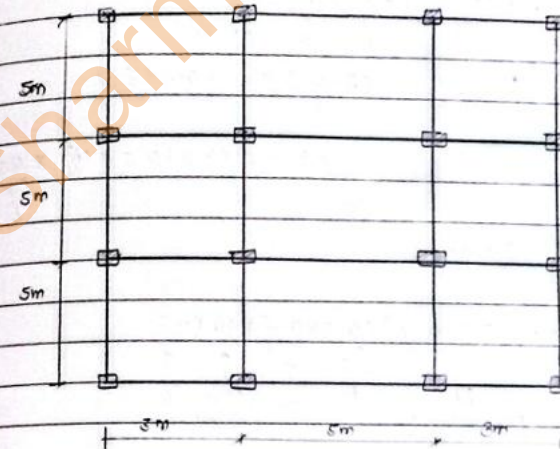
* This design lateral force (base shear) shall be distributed along height of building at each storey level as per Cl. 7.6.3(a) of IS 1893: 2016 given by

$$Q_i = V_B \times \frac{W_i h_i^2}{\sum W_i h_i^2}$$

Storey	Height (h_i) (m)	Seismic wt. (W_i) (kN)	$W_i h_i^2$	Q_i (kN)	Storey shear (V_i), kN
5	15	3960	891000	1156.56	1156.56
4	12	3960	570240	740.16	1896.66
3	9	3960	320760	416.34	2313.00
2	6	3960	142560	185.04	2498.04
1	3	3960	35640	46.26	2544.30
$\sum W_i h_i^2 = 1960200$					



9. The figure shows a plan of 5-storey RC framed structure to be constructed in Pothan area. The height of each storey is 3.5m, all columns are 500mm x 500mm and all beams are 300mm x 500mm. The slab thickness is 150mm. The masonry wall is 230mm thick brick wall. The soil below foundation is soft. Assume the live load on all floors is 3 kN/m^2 and on the roof is 2 kN/m^2 . Determine the lateral seismic force at different floor levels. Assume relevant data if required as per NBC 105: 2020.



→ Soln:

Soil condition = soft

Ht. of each storey = 3.5m

No. of storey = 5

Column: 500mm x 500mm

Beam: 300mm x 500mm

Slab thickness = 150mm

Masonry wall thk = 230mm

LL on floor = 3 kN/m^2

LL on Roof = 2 kN/m^2

* Calculation of seismic weight of each floor
[Cl. 5.2, NBC 105:2020]

* For 1st floor

Dead Load

$$\text{Slab} = \underbrace{(15+0.5)}_L * \underbrace{(11+0.5)}_B * \underbrace{0.15}_t * \underbrace{25}_{\gamma_c}$$

$$= 668.4375 \text{ kN}$$

$$\text{Beam} = \underbrace{\frac{1}{2} (5-0.5) * 0.3 * 0.5 * 12}_{\substack{\text{(deducting)} \\ \text{column}}} + \underbrace{(3-0.5) * 0.3 * 0.5 * 8}_B + \underbrace{(5-0.5) * 0.3 * 0.5 * 4}_{\substack{\text{(deducting)} \\ \text{column}}} * 25$$

$$= 345 \text{ kN}$$

$$\text{Column} = \underbrace{(3.5-0.15)}_H * \underbrace{0.5}_B * \underbrace{0.5}_L * \underbrace{16}_{\substack{\text{Nos.} \\ \text{(nodes)}}} * \underbrace{25}_{\gamma_c}$$

$$= 335 \text{ kN}$$

$$\text{Masonry wall} = \underbrace{\frac{1}{2} (5-0.5) * 0.23}_{\substack{\text{(deducting)} \\ \text{column}}} * \underbrace{(3.5-0.5)}_B * \underbrace{16}_{\substack{\text{Nos.} \\ \text{(deducting beam)}}} + (3-0.5) * 0.23 * (3.5-0.5) * 8$$

$$+ \underbrace{20}_{\gamma_{\text{masonry}}} * \underbrace{0.8}_{\substack{\text{thickness} \\ \text{door opening}}}$$

$$= 1015.68 \text{ kN}$$

$$\therefore \text{Total dead load} = 668.4375 + 345 + 335 + 1015.68$$

$$= 2364.20 \text{ kN}$$

$$\text{Live load} = \underbrace{(15.5 * 11.5)}_{\substack{\text{Total dimension} \\ \text{including} \\ \text{column projection}}} * \underbrace{(0.3 * 3)}_{\substack{\text{Table 5-1} \\ \text{For other purpose}}}$$

$$= 160.425 \text{ kN}$$

$$\therefore \text{Seismic wt. of 1st floor, } W_1 = 2364.20 + 160.425$$

$$= 2524.65 \text{ kN}$$

Similarly,

$$W_2 = W_3 = W_4 = 2524.65 \text{ kN (similar floors)}$$

* For roof (5th floor)

Dead load

$$\text{Slab} = 668.4375 \text{ kN}$$

$$\text{Beam} = 345 \text{ kN}$$

$$\text{Column} = \underbrace{\left(\frac{3.5}{2} - 0.15 \right)}_H * 0.5 * 0.5 * 16 * 25$$

$$= 160 \text{ kN}$$

$$\text{Masonry wall} = \frac{1}{2} * 2 * 15.5 + 2 * (11.5 - 0.23) * 0.23$$

$$* 0.9 * 20$$

(assume 0.9m high parapet wall all around the roof)

$$= 221.6556 \text{ kN}$$

$$\therefore \text{Total dead load} = 668.4375 + 345 + 160 + 221.6556$$

$$= 1395.10 \text{ kN}$$

$$\text{Live load} = (15.5 * 11.5) * 2 * \frac{0}{\text{Table (5-1)}}$$

$$= 0$$

∴ seismic wt. of roof; $W_r = 1395.10 \text{ kN}$

* Total seismic weight, $W = W_1 + W_2 + W_3 + W_4 + W_r$

$$= 2524.65 \times 4 + 1395.10$$

$$= 11,493.70 \text{ kN}$$

* Fundamental time period $\{ \text{Cl. 5.1.2} \} \& \{ \text{Cl. 5.1.3} \}$ considering MRF,

$$T_1 = \underbrace{0.075}_{\substack{\uparrow \\ \text{(Cl. 5.1.2)}}} \times \underbrace{H^{0.75}}_{\substack{\uparrow \\ \text{(Cl. 5.1.3)}}} \times \underbrace{1.25}_{\substack{\uparrow \\ \text{(H=5 \times 3.5=17.5)}}}$$

$$= 0.075 \times (17.5)^{0.75} \times 1.25$$

$$\approx 0.802 \text{ sec}$$

* Now, design horizontal base shear coefficient, $C_d(T_1)$

$$C_d(T_1) = \frac{C(T_1)}{R_\mu \times S_u} \quad \{ \text{Cl. 6.1.1} \}$$

where,

$$C(T_1) = C_h(T_1) \cdot Z \cdot I \quad \{ \text{Cl. 4.1.1} \}$$

$$C_h(T_1) = 2.25 \quad \{ \text{Cl. 4.1.2} \& \text{Table 4.1} \}$$

$$Z = 0.35 \quad \text{for Lalitpur Paton area} \\ \{ \text{Cl. 4.1.4; NBC 105} \}$$

$$I = 1.25 \quad \{ \text{Cl. 4.1.5, NBC 105} \}$$

(assuming importance class II)

$$\therefore C(T_1) = 2.25 \times 0.35 \times 1.25$$

$$= 0.984375$$

$$R_\mu = \text{ductility factor (Cl. 5.3; Table 5.2)}$$

$$= 4 \quad \text{for RC MRF}$$

$$S_u = \text{overstrength factor (Cl. 5.4, Table 5.2)}$$

$$= 1.5 \quad \text{for RC MRF}$$

$$\therefore C_d(T_1) = \frac{0.984375}{4 \times 1.5} = 0.1640625$$

* Horizontal seismic base shear (Cl. 6.2)

$$V = C_d(T_1) \times W$$

$$= 0.1640625 \times 11,493.70$$

$$\approx 1885.70 \text{ kN}$$

* Horizontal seismic base shear shall be distributed along height of building as per Cl. 6.3 of NBC 105: 2020 is given by

$$F_i = \frac{W_i h_i^k}{\sum W_i h_i^k} \times V$$

where,

$$k = \frac{1 + (2-1)}{(2.5-0.5)} \times (0.802 - 0.5)$$

(for $0.5 \leq T_1 \leq 2.5$)

$$\therefore k = 1.551$$

$$\therefore F_i = \frac{W_i h_i^{\frac{1}{2}}}{\sum W_i h_i^{\frac{1}{2}}} \times V$$

Storey	height (h _i) (m)	storey weight (W _i), kN	W _i h _i ^{1/2}	F _i (kN)	Storey shear (V _i), kN
5	17.5	1285.10	37613.36	436.594	436.594
4	14.0	2524.65	52649.54	611.125	1047.719
3	10.5	2524.65	37808.55	438.853	1486.572
2	7.0	2524.65	23708.77	275.197	1761.775
1	3.5	2524.65	10676.37	123.925	1885.700

$$\sum W_i h_i^{\frac{1}{2}} = 162,456.53$$

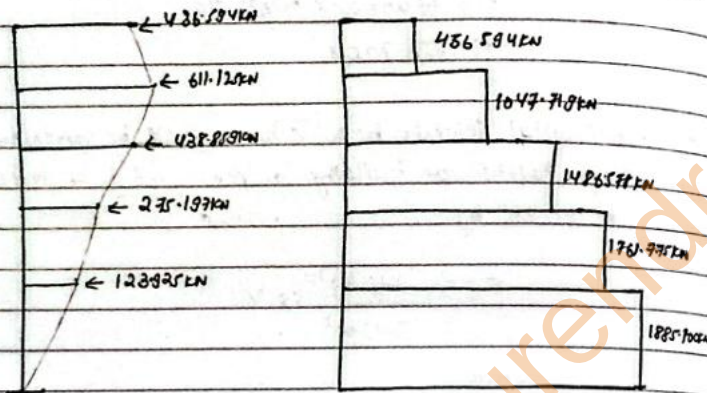


Fig: lateral force distribution

Fig: Storey shear distribution

Mode Participation factor
For Mode k,

$$\lambda_k = \frac{L_k}{M_k} = \frac{\{\phi_k\}^T [m] \{f\}}{\{\phi_k\}^T [m] \{\phi_k\}} = \frac{\sum m_i \phi_{ik}}{\sum m_i (\phi_{ik})^2} \times \frac{g}{g} = \frac{\sum W_i \phi_{ik}}{\sum W_i (\phi_{ik})^2}$$

Effective modal mass (part of total mass responding to particular mode, k)

$$M_k^* = \frac{L_k^2}{M_k} = \frac{(\{\phi_k\}^T [m] \{f\})^2}{\{\phi_k\}^T [m] \{\phi_k\}} = \frac{(\sum W_i \phi_{ik})^2}{g \sum W_i (\phi_{ik})^2}$$

Peak storey force (Max^m elastic force) in mode, k,

* Maximum/peak base shear in mode, k

$$V_{b,k} = \frac{L_k^2}{M_k} \times S_{a,k} = M_k^* \times S_{a,k} = M_k^* \cdot g \cdot \frac{S_{a,k}}{g}$$

$$[V_{b,k} = W_k^* \cdot A_k]$$

$$W_k^* = M_k^* \cdot g = \text{Effective modal weight}$$

$$A_k = \frac{S_{a,k}}{g} = \text{acceleration spectrum for mode 'k' normalized with 'g'}$$

* Peak storey force (Max^m elastic force)

$$\{F_k\} = \{F_k\} = [m] \{F_k\} \cdot \frac{L_k}{M_k} \cdot S_{a,k}$$

For particular storey 'i' in mode 'k'

$$F_{ik} = F_{ik} = m_i \phi_{ik} \cdot \frac{L_k}{M_k} \cdot S_{a,k} \quad \text{--- (i)}$$

$$= m_i \phi_{ik} \cdot \lambda_k \cdot S_{a,k} \cdot \frac{g}{g}$$

$$= \frac{S_{a,k}}{g} \cdot \phi_{ik} \cdot \lambda_k \cdot W_i$$

$$[F_{ik} = F_{ik} = A_k \cdot \phi_{ik} \cdot \lambda_k \cdot W_i]$$

From (1)

$$S_{ik} = F_{ik} = m_i \phi_{ik} \cdot \frac{L_k}{M_k} \cdot \frac{L_k}{L_k} \cdot S_{0,k}$$

$$= m_i \phi_{ik} \cdot \frac{L_k^2}{M_k} \cdot \frac{1}{L_k} \cdot S_{0,k}$$

$$= \frac{S_{0,k}}{g} \cdot \underbrace{g m_i \phi_{ik}}_{w_i \phi_{ik}} \cdot \underbrace{\frac{M_k^* \cdot g}{g \cdot L_k}}_{w_k^*} \cdot \frac{1}{g \cdot L_k}$$

$$= A_k \cdot w_i \phi_{ik} \cdot w_k^* \cdot \frac{1}{g \sum w_i \phi_{ik}}$$

$$\left[\phi_{ik} = \underbrace{(A_k \cdot w_k^*)}_{V_{b,ik}} \cdot \frac{w_i \phi_{ik}}{\sum w_i \phi_{ik}} \right]$$

Effective Modal mass ratio (or γ)
or

Max participating ratio

$$\sum M_k^* = \sum_{k=1}^n \frac{L_k^2}{M_k} = \sum w_i = \text{total mass of structure}$$

ratio of total mass participating in mode 'k'

$$\left[\gamma_k = \frac{M_k^*}{\sum M_k^*} = \frac{w_k^*}{\sum w_i} \times 100\% \right]$$

Normalized accelⁿ spectrum (A_k)
VS

Design hz. seismic coeff. as per IS 1893:2016
(acceleration spectrum) CCI: 6.4.2)

$$\left\{ A_k = \frac{S_{0,k}}{g} \right.$$

$$\left. \text{VS} \int A_{h,ik} = \frac{(Z/a)}{R/I} \left(\frac{S_{0,k}}{g} \right) \right.$$

A_k → doesn't include info about

→ seismic intensity of the region

↳ Importance of structure

↳ ductility & energy dissipation capacity of SD

In practical problem use, $A_{h,k}$ instead of A_k .

Normalized acceleration spectrum (A_k)

VS
Design hz. base shear coeff. as per NBC 105:2020
(CI: 7.1)

$$A_k = \frac{S_{0,k}}{g}$$

VS

$$C_d(T_k) = \frac{C(T_k)}{R_M \times R_u} = \underbrace{C_h(T_k)}_{\text{spectral shape factor}} \times \left(\frac{Z \cdot I}{R_M \times R_u} \right)$$

spectral shape factor
equiv to $\frac{S_{0,k}(T)}{g}$

(9) For the three storey building frame, the seismic weight on floors are: weight of first floor (W_1) = 1176 kN, weight of second floor (W_2) = 784 kN, weight of third floor (W_3) = 392 kN. The building is founded on medium stiff soil and situated in zone V according to IS 1893:2016 and considering $Z=0.36$, $I=1$, $R=5$.

The free vibration results are: natural periods

$$\{T_n\}^T = [0.883 \ 0.404 \ 0.302]$$

and mode shapes are

$$\{\phi_1\}^T = [0.25 \ 0.79 \ 1]$$

$$\{\phi_2\}^T = [-1 \ 0 \ 1]$$

$$\{\phi_3\}^T = [0.25 \ -0.79 \ 1]$$

Determine the seismic force by dynamic analysis procedure codal provisions.

→ Solⁿ:

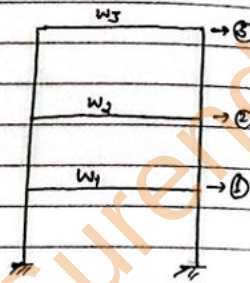
$$\{T_n\} = \begin{Bmatrix} 0.883 \\ 0.404 \\ 0.302 \end{Bmatrix}$$

$$[Q] = \begin{bmatrix} 0.25 & -1 & 0.25 \\ 0.79 & 0 & -0.79 \\ 1 & 1 & 1 \end{bmatrix}$$

$$Z=0.36$$

$$I=1$$

$$R=5$$



* Calculation of

Modal participation factor

$$\lambda_k = \frac{\sum W_i \phi_{ik}}{\sum W_i (\phi_{ik})^2} \quad (7.7.5.4. (a))$$

Effective modal mass

$$M_k^* = \frac{(\sum W_i \phi_{ik})^2}{g \sum W_i (\phi_{ik})^2} \quad (7.7.5.4. (b))$$

Storey (i)	Weight (W_i) kN	Mode 1			Mode 2			Mode 3		
		ϕ_{i1}	$W_i \phi_{i1}$	$W_i \phi_{i1}^2$	ϕ_{i2}	$W_i \phi_{i2}$	$W_i \phi_{i2}^2$	ϕ_{i3}	$W_i \phi_{i3}$	$W_i \phi_{i3}^2$
1	1176	0.25	294	73.5	-1	-1176	1176	0.25	294	73.5
2	784	0.79	619.36	489.294	0	0	0	-0.79	-619.36	489.294
3	392	1	392	392	1	392	392	1	392	392
Sum (Σ)			1305.36	954.734		-784	1568		66.706	954.734
Modal participation factor (λ_k)		$\lambda_1 = \frac{1305.36}{954.734} = 1.367$			$\lambda_2 = \frac{-784}{1568} = -0.5$			$\lambda_3 = \frac{66.706}{954.734} = 0.070$		
Effective Modal mass (M_k^*)		$M_1^* = \frac{(1305.36)^2}{9.81 \times 954.734} = 181.92 \times 10^3 \text{ kg}$			$M_2^* = \frac{(-784)^2}{9.81 \times 1568} = 38.85 \times 10^3 \text{ kg}$			$M_3^* = \frac{(66.706)^2}{9.81 \times 954.734} = 0.475 \times 10^3 \text{ kg}$		
% of total mass		$\gamma_1 = 81.81\%$			$\gamma_2 = 17.97\%$			$\gamma_3 = 0.21\%$		

As per IS 1893, the number of modes to be considered such that at least 90% of mass to be included; but for higher accuracy, let us consider all modes. Now, let us calculate storey force due to each of mode using,

$$F_{ik} = A_{h,k} \times \phi_{ik} \times \lambda_k \times W_i$$

where, $A_{h,k} = \frac{S_{a,k}}{g} \times \left(\frac{Z}{S}\right) \times \frac{I}{R}$

Given:
 $Z=0.36$
 $I=1$
 $R=5$

Mode 1	Mode 2	Mode 3
$T_1 = 0.883$	$T_2 = 0.404$	$T_3 = 0.302$
For medium stiff soil		
$\frac{S_{a,1}}{g} = \frac{1.36}{0.883} = 1.54$	$\frac{S_{a,2}}{g} = 2.5$	$\frac{S_{a,3}}{g} = 2.5$
$A_{h,1} = 0.05514$	$A_{h,2} = 0.09$	$A_{h,3} = 0.09$

The storey force/lateral force and storey shear calculations in each mode are shown in table below:

Storey (i)	Weight (W _i), kN	Mode 1			Mode 2			Mode 3		
		Φ_{i1}	Φ_{i2}	Φ_{i3}	Φ_{i1}	Φ_{i2}	Φ_{i3}	Φ_{i1}	Φ_{i2}	Φ_{i3}
3	382	1	29.708	29.708	0.1	-17.64	1	2.469	2.469	
2	784	0.79	46.939	76.647	0	0	-0.79	3.902	6.371	
1	1176	0.25	22.291	98.920	-1	35.28	0.25	1.851	8.223	

The combined storey shear using SRS

$$V_{top} = V_3 = \sqrt{29.708^2 + (-17.64)^2 + (2.469)^2} = 34.639 \text{ kN}$$

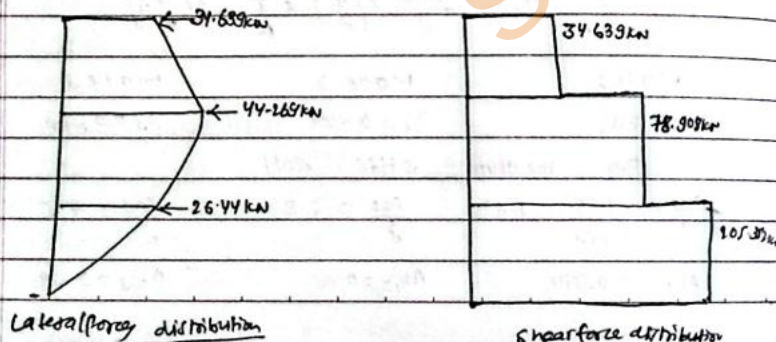
Similarly,

$$V_2 = 78.908 \text{ kN}$$

$$V_1 = 105.374 \text{ kN}$$

∴ The lateral forces at each storey due to all mode considered (CI-7-7-5.4(f); IS 1893:2016) are

$$\begin{aligned} F_{top} &= F_3 = 34.639 \text{ kN} \\ F_2 &= V_2 - V_3 = 44.269 \text{ kN} \\ F_1 &= V_1 - V_2 = 26.444 \text{ kN} \end{aligned}$$

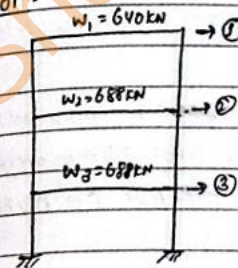


Q. Determine the design seismic force for a residential building in Kathmandu using modal response spectrum as per NBC 105. Also show the distribution of lateral forces with building height. Take building frame as SRF.

$$\{W\} = \begin{Bmatrix} 11.79 \\ 32.93 \\ 47.04 \end{Bmatrix} \text{ kN}; \quad [W] = \begin{bmatrix} 640 & 0 & 0 \\ 0 & 688 & 0 \\ 0 & 0 & 688 \end{bmatrix} \text{ kN};$$

$$[Q] = \begin{bmatrix} 1 & 1 & 1 \\ 0.81 & -0.489 & -2.038 \\ 0.45 & -1.223 & 1.611 \end{bmatrix}$$

→ Soln:



$$\{W_k\} = \begin{Bmatrix} 11.79 \\ 32.93 \\ 47.04 \end{Bmatrix} \text{ kN} \Rightarrow [T_k] = \begin{Bmatrix} 0.533 \\ 0.181 \\ 0.136 \end{Bmatrix}$$

$$T_k = \frac{2\pi}{\omega_k}$$

* Calculation of effective modal weight

$$CI-7-2 \quad \{W_k^*\} = \frac{(\sum W_i \Phi_{ik})^2}{\sum W_i (\Phi_{ik})^2}$$

⇒ Modal base shear

$$CI-7-2 \quad \{V_{bk}\} = C_d(T_k) \times W_k^*$$

⇒ Modal storey force

$$CI-7-2 \quad \{F_{ik}\} = V_{bk} \times \frac{W_i \Phi_{ik}}{\sum W_i \Phi_{ik}}$$

⇒ Modal weight participating ratio percentage

$$\eta_k = \frac{W_k^*}{\sum W_k^*} \times 100\%$$

First, let us calculate effective modal weight and Modal wt ratio:

Storey (i)	Weight (W _i) (kN)	Mode 1			Mode 2			Mode 3		
		φ _{i,1}	W _i φ _{i,1}	W _i φ _{i,1} ²	φ _{i,2}	W _i φ _{i,2}	W _i φ _{i,2} ²	φ _{i,3}	W _i φ _{i,3}	W _i φ _{i,3} ²
1	640	1	640	640	1	640	640	1	640	640
2	682	0.81	527.22	427.40	-0.483	-326.43	164.52	-2.238	-1402.17	2227.77
3	682	0.45	309.60	139.32	-1.223	-841.42	1029.06	1.611	1108.37	1785.57
Sum (Σ)	2016		1506.88	1230.72		-537.85	1883.57		346.23	5223.15
Effective Modal Weight (W _{ek})		$W_{e1} = \frac{(1506.88)^2}{1230.72} = 1845.01 \text{ kN}$			$W_{e2} = \frac{(-537.85)^2}{1883.57} = 151.77 \text{ kN}$			$W_{e3} = \frac{(346.23)^2}{5223.15} = 22.69 \text{ kN}$		
Modal weight ratio (α _i)		α ₁ = 91.03%			α ₂ = 7.73%			α ₃ = 1.12%		

② As per NBC 105, the number of modes shall be included in the analysis to include at least 90% of total seismic weights, which is satisfied here by only 1st mode. But, for higher accuracy, let us consider all modes.

For given building (SMRF); from NBC 105; we have
 $\mu = 0.35$ (for Kathmandu area)
 $I = 1$ (for ordinary residential building)
 $R_u = 4$ for RC SMRF building
 $R_u = 1.5$

Taking Soil type D → for Kathmandu region

Let us calculate horizontal base shear coefficient for each mode 'k' using

$$C_d(T_k) = \frac{C_u(T_k)}{R_u \cdot R_w} = \frac{C_u(T_k)}{R_u \cdot \alpha_k} = \frac{C_u(T_k)}{1.5 \cdot \alpha_k} = C_u(T_k) \cdot 0.6667$$

Mode 1

$$T_1 = 0.533 \text{ sec}$$

For soil type D,

Mode 2

$$T_2 = 0.194 \text{ sec}$$

Mode 3

$$T_3 = 0.136 \text{ sec}$$

$$T_a = 0.5$$

$$T_c = 0.2$$

$$T_d = 5$$

$$\alpha = 2.25$$

(Table 4-1)

[C1.4.1.2]

$$C_h(T_1) = \alpha$$

$$(\because T_a < T_1 < T_c)$$

$$\therefore C_h(T_1) = 2.25$$

$$C_h(T_2) =$$

$$1 + (\alpha - 1) \frac{T_a}{T_2} \quad (\because T_a < T_2)$$

$$\therefore C_h(T_2) = 1.4775$$

$$C_h(T_3) = 1 + (\alpha - 1) \frac{T_a}{T_3}$$

$$(\because T_3 < T_a)$$

$$\therefore C_h(T_3) = 1.34$$

Hence,

$$C_d(T_1) = 2.25 \times 0.6667 = 1.5$$

$$C_d(T_2) = 1.4775 \times 0.6667 = 0.985$$

$$C_d(T_3) = 1.34 \times 0.6667 = 0.893$$

Also,

$$V_{b1} = C_d(T_1) \times W_{e1} = 1.5 \times 1845.01 = 2767.52 \text{ kN}$$

Also,

$$V_{b2} = C_d(T_2) \times W_{e2} = 0.985 \times 151.77 = 149.5 \text{ kN}$$

Also,

$$V_{b3} = C_d(T_3) \times W_{e3} = 0.893 \times 22.69 = 20.25 \text{ kN}$$

Now, the modal storey force and modal storey shear calculation for each mode are shown in table below (using 1st & 2nd table),

Storey (i)	Mode 1			Mode 2			Mode 3		
	W _i φ _{i,1}	F _{i,1} (kN)	V _{i,1} (kN)	W _i φ _{i,2}	F _{i,2} (kN)	V _{i,2} (kN)	W _i φ _{i,3}	F _{i,3} (kN)	V _{i,3} (kN)
1	640	102.85	102.85	640	-16.18	-16.18	640	3.28	3.28
2	527.22	89.56	192.41	-326.43	8.50	-7.68	-1402.17	-7.18	-3.90
3	309.60	48.75	242.16	-841.42	21.27	13.59	1108.37	15.568	1.78
Sum	1506.88	242.16		-537.85	12.59		246.23	24.72	

$$\alpha = 7.1(3)$$

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The combined storey shear due to all modes using SRS:

$$V_{top} = V_1 = \sqrt{102.85^2 + (-16.18)^2 + (3.28)^2} = 104.17 \text{ kN}$$

$$V_2 = \sqrt{192.41^2 + (-7.68)^2 + (-3.90)^2} = 192.60 \text{ kN}$$

$$V_{base} = V_3 = \sqrt{(242.16)^2 + (13.59)^2 + (1.72)^2} = 242.55 \text{ kN}$$

Now, the lateral forces due to all modes at each storey using algebraic summation are:

$$F_{top} = F_1 = 104.17 \text{ kN}$$

$$F_2 = V_2 - V_1 = 88.43 \text{ kN}$$

$$F_3 = V_3 - V_2 = 49.95 \text{ kN}$$

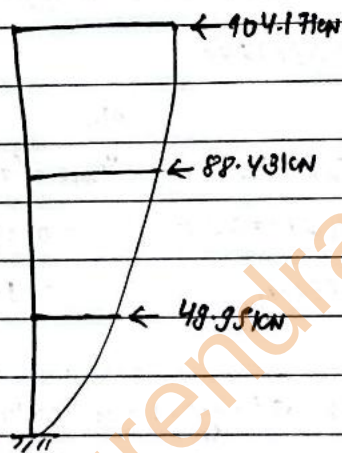


Fig: Lateral force distribution

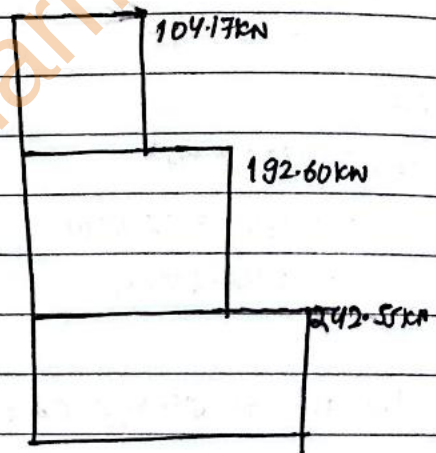


Fig: Shear force distribution